LIMIT DEFINITION

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

7. $f(x,y) = x^2 - 3y$

$f_x(x,y)$

① $f(\underset{x}{x+\Delta x}, \underset{y}{y}) = (x+\Delta x)^2 - 3(y)$

$= (x+\Delta x)(x+\Delta x) - 3y$

$= x^2 + x\Delta x + x\Delta x + (\Delta x)^2 - 3y$

$= x^2 + 2x\Delta x + (\Delta x)^2 - 3y$

② $f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 3y - (x^2 - 3y)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 3y - x^2 + 3y}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x (2x + \Delta x)}{\Delta x}$

$= \lim_{\Delta x \to 0} 2x + \Delta x$

$= 2x + 0$

$= 2x$

$f_y(x,y)$

① $f(\underset{x}{x}, \underset{y}{y+\Delta y}) = (x)^2 - 3(y+\Delta y)$

$= x^2 - 3y - 3\Delta y$

② $f_y(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$

$= \lim_{\Delta y \to 0} \frac{x^2 - 3y - 3\Delta y - (x^2 - 3y)}{\Delta y}$

$= \lim_{\Delta y \to 0} \frac{x^2 - 3y - 3\Delta y - x^2 + 3y}{\Delta y}$

$= \lim_{\Delta y \to 0} \frac{-3\Delta y}{\Delta y}$

$= \lim_{\Delta y \to 0} -3$

$= -3$