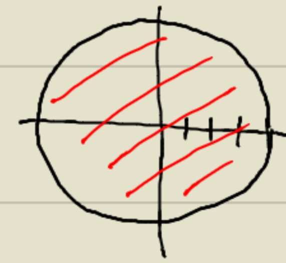
4. $f(x,y) = \sqrt{x^2+y^2} = (x^2+y^2)^{\frac{1}{2}}$

$R = \{(x,y) : 0 \leq f(x,y) \leq 4\}$



$f_x = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x$

$= \frac{x}{(x^2+y^2)^{1/2}} = \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y$

$= \frac{y}{(x^2+y^2)^{1/2}} = \frac{y}{\sqrt{x^2+y^2}}$

$0 \leq \sqrt{x^2+y^2} \leq 4$

$\sqrt{x^2+y^2} = 4$

$x^2+y^2 = 16$

$$S = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=4} \left(\sqrt{1 + \left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2}\; r \right) dr\, d\theta$$

$$S = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=4} \left(\sqrt{1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}}\; r \right) dr\, d\theta$$

$$S = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=4} \left(\sqrt{1+1}\; r \right) dr\, d\theta$$

$$S = \sqrt{2} \int_{\theta=0}^{\theta=2\pi} \left[\frac{1}{2}r^2\right]_{r=0}^{r=4} d\theta$$

$$S = \frac{\sqrt{2}}{2} \int_{\theta=0}^{\theta=2\pi} \left[4^2 - 0^2\right] d\theta$$

$$S = \frac{16\sqrt{2}}{2} \int_{\theta=0}^{\theta=2\pi} d\theta$$

$$S = 8\sqrt{2}\left[\theta\right]_{\theta=0}^{\theta=2\pi}$$

$$S = 8\sqrt{2}\left[2\pi - 0\right]$$

$$S = 16\pi\sqrt{2}$$