5. $\iint_S f(x,y)\,dS$ $\quad f(x,y) = x+y$ $\quad \vec{r}(u,v) = \underbrace{(8\cos u)}_{x(u,v)}\vec{i} + \underbrace{(8\sin u)}_{y(u,v)}\vec{j} + \underbrace{v}_{z(u,v)}\vec{k}$

$0 \le u \le \frac{\pi}{2} \qquad 0 \le v \le 4$

$$= \int_{v=0}^{v=4}\int_{u=0}^{u=\frac{\pi}{2}} (8\cos u + 8\sin u)\cdot 8 \, du\, dv$$

$$= 64 \int_{v=0}^{v=4}\int_{u=0}^{u=\frac{\pi}{2}} (\cos u + \sin u)\, du\, dv$$

$$= 64 \int_{v=0}^{v=4} \Big[\sin u - \cos u\Big]_{u=0}^{u=\frac{\pi}{2}} dv$$

$$= 64 \int_{v=0}^{v=4} \left[\sin\tfrac{\pi}{2} - \cos\tfrac{\pi}{2} - (\sin 0 - \cos 0)\right] dv$$

$$= 64 \int_{v=0}^{v=4} \Big[1 - 0 - (0-1)\Big] dv$$

$$= 64 \cdot 2 \int_{v=0}^{v=4} dv$$

$$= 128 \Big[v\Big]_{v=0}^{v=4}$$

$$= 128\,[4 - 0]$$

$$= \boxed{512}$$

$f[x(u,v), y(u,v)]$

$= f(8\cos u, 8\sin u) = 8\cos u + 8\sin u$

$\vec{r}_u = -8\sin u\,\vec{i} + 8\cos u\,\vec{j}$

$\vec{r}_v = \vec{k}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8\sin u & 8\cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \begin{matrix} (+) & (-) & (+) \end{matrix}$$

$$= \vec{i}\begin{vmatrix} 8\cos u & 0 \\ 0 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} -8\sin u & 0 \\ 0 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} -8\sin u & 8\cos u \\ 0 & 0 \end{vmatrix}$$

$= 8\cos u\,\vec{i} + 8\sin u\,\vec{j}$

$$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{(8\cos u)^2 + (8\sin u)^2}$$

$$= \sqrt{64\cos^2 u + 64\sin^2 u}$$

$$= \sqrt{64(\cos^2 u + \sin^2 u)}$$

$= 8$