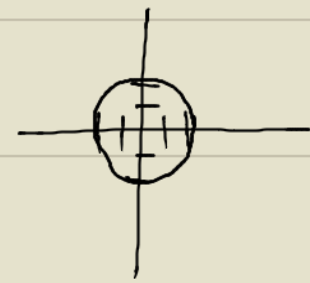
6. $\iint_S f(x,y,z)\,dS \qquad f(x,y,z)=x^2+y^2+z^2 \qquad S: z=x+y,\ x^2+y^2\le 4$

$= \iint_S (x^2+y^2+z^2)\,dS \qquad g(x,y)=x+y$

$= \iint_S (x^2+y^2+(x+y)^2)\,dS \qquad g_x=1 \quad g_y=1$



$= \iint_S (x^2+y^2+x^2+2xy+y^2)\,dS$

$= \iint (2x^2+2y^2+2xy)\,dS$

$= 2\iint (x^2+y^2+xy)\,dS \qquad r^2=x^2+y^2$

$= 2\iint (x^2+y^2+xy)\sqrt{1+(1)^2+(1)^2} \qquad x=r\cos\theta \quad y=r\sin\theta$

$= 2\sqrt{3}\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2} (r^2+r\cos\theta\, r\sin\theta)\,r\,dr\,d\theta$

$= 2\sqrt{3}\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2} (r^3+r^3\cos\theta\sin\theta)\,dr\,d\theta$

$= 2\sqrt{3}\int_{\theta=0}^{\theta=2\pi} \left[\frac{1}{4}r^4+\frac{1}{4}r^4\cos\theta\sin\theta\right]_{r=0}^{r=2} d\theta$

$= 2\sqrt{3}\cdot 4\int_{\theta=0}^{\theta=2\pi} (1+\cos\theta\sin\theta)\,d\theta$

$u=\sin\theta \quad du=\cos\theta\,d\theta$

$= 8\sqrt{3}\left[\theta+\frac{1}{2}\sin^2\theta\right]_{\theta=0}^{\theta=2\pi}$

$= 8\sqrt{3}\left[2\pi+\frac{1}{2}\sin^2(2\pi)-\left(0+\frac{1}{2}\sin^2 0\right)\right]$

$= 8\sqrt{3}(2\pi)$

$= 16\sqrt{3}\pi$