

$$1. \vec{r}(t) = 5t\vec{i} + t^3\vec{j}, \quad t=1$$

$$\vec{r}'(t) = 5\vec{i} + 3t^2\vec{j}$$

$$\vec{r}'(1) = 5\vec{i} + 3(1)^2\vec{j}$$

$$\vec{r}'(1) = 5\vec{i} + 3\vec{j}$$

$$\|\vec{r}'(1)\| = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

$$\begin{aligned} \vec{T}(1) &= \frac{\vec{r}'(1)}{\|\vec{r}'(1)\|} = \frac{5\vec{i} + 3\vec{j}}{\sqrt{34}} = \frac{5}{\sqrt{34}}\vec{i} + \frac{3}{\sqrt{34}}\vec{j} \\ &= \boxed{\frac{5\sqrt{34}}{34}\vec{i} + \frac{3\sqrt{34}}{34}\vec{j}} \end{aligned}$$

$$2. \vec{r}(t) = 2\cos t\vec{i} + 3\sin t\vec{j}, \quad t = \frac{\pi}{6}$$

$$\vec{r}'(t) = -2\sin t\vec{i} + 3\cos t\vec{j}$$

$$\vec{r}'\left(\frac{\pi}{6}\right) = -2\sin\frac{\pi}{6}\vec{i} + 3\cos\frac{\pi}{6}\vec{j}$$

$$= -2\left(\frac{1}{2}\right)\vec{i} + 3\left(\frac{\sqrt{3}}{2}\right)\vec{j}$$

$$\vec{r}'\left(\frac{\pi}{6}\right) = -\vec{i} + \frac{3\sqrt{3}}{2}\vec{j}$$

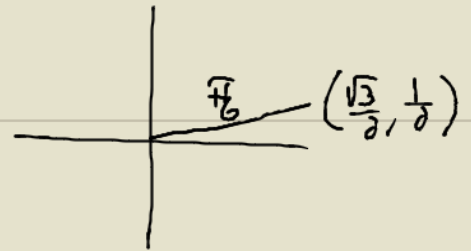
$$\|\vec{r}'\left(\frac{\pi}{6}\right)\| = \sqrt{(-1)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{1 + \frac{27}{4}}$$

$$= \sqrt{\frac{31}{4}}$$

$$= \frac{\sqrt{31}}{2}$$

$$= \boxed{\frac{\sqrt{31}}{2}}$$



$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$

$$= \frac{-\vec{i} + \frac{3\sqrt{3}}{2}\vec{j}}{\frac{\sqrt{31}}{2}}$$

$$= \frac{-2}{\sqrt{31}}\vec{i} + \frac{2}{\sqrt{31}} \cdot \frac{3\sqrt{3}}{2}\vec{j}$$

$$= \frac{-2\sqrt{31}}{31}\vec{i} + \frac{3\sqrt{3}}{\sqrt{31}}\vec{j}$$

$$= \boxed{\frac{-2\sqrt{31}}{31}\vec{i} + \frac{3\sqrt{93}}{31}\vec{j}}$$