

$$7. \vec{r}(t) = \ln(t) \vec{i} + t^2 \vec{j} + 5t \vec{k}, \quad t=1$$

$$\vec{r}'(t) = \frac{1}{t} \vec{i} + 2t \vec{j} \quad \|\vec{r}'(t)\| = \sqrt{\left(\frac{1}{t}\right)^2 + (2t)^2}$$

$$\hat{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$

$$= \frac{\frac{1}{t} \vec{i} + 2t \vec{j}}{\sqrt{\frac{4t^4+1}{t^2}}}$$

$$= \frac{t}{\sqrt{4t^4+1}} \cdot \frac{1}{t} \vec{i} + \frac{t}{\sqrt{4t^4+1}} \cdot 2t \vec{j}$$

$$\hat{T}(t) = (4t^4+1)^{-\frac{1}{2}} \vec{i} + \frac{2t^2}{(4t^4+1)^{\frac{1}{2}}} \vec{j}$$

$$P = 4t^4 \quad Q = (4t^4+1)^{-\frac{1}{2}} \cdot (16t^3)$$

$$= \frac{-8t^3}{(4t^4+1)^{\frac{3}{2}}}$$

$$P'Q + PQ'$$

$$\hat{T}'(t) = \frac{-8t^3}{(4t^4+1)^{\frac{3}{2}}} \vec{i} + \left( 4t(4t^4+1)^{-\frac{1}{2}} + 2t^2 \left( \frac{-8t^3}{(4t^4+1)^{\frac{3}{2}}} \right) \right) \vec{j}$$

$$\hat{T}'(1) = \frac{-8}{5^{\frac{3}{2}}} \vec{i} + \left( 4(5)^{-\frac{1}{2}} + 2 \left( \frac{-8}{5^{\frac{3}{2}}} \right) \right) \vec{j}$$

$$= \frac{-8}{5\sqrt{5}} \vec{i} + \left( \frac{4}{5^{1/2}} - \frac{16}{5^{3/2}} \right) \vec{j}$$

$$= \frac{-8}{5\sqrt{5}} \vec{i} + \left( \frac{20}{5^{3/2}} - \frac{16}{5^{3/2}} \right) \vec{j}$$

$$\hat{T}'(1) = \frac{-8}{5\sqrt{5}} \vec{i} + \frac{4}{5\sqrt{5}} \vec{j}$$

$$\|\hat{T}'(1)\|$$

$$= \sqrt{\left(\frac{-8}{5\sqrt{5}}\right)^2 + \left(\frac{4}{5\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{80}{125}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\hat{N}(1) = \frac{\hat{T}'(1)}{\|\hat{T}'(1)\|} = \frac{\frac{-8}{5\sqrt{5}} \vec{i} + \frac{4}{5\sqrt{5}} \vec{j}}{\frac{4}{5}} = \frac{-2}{\sqrt{5}} \vec{i} + \frac{1}{\sqrt{5}} \vec{j} = \left( \frac{-2\sqrt{5}}{5} \vec{i} + \frac{\sqrt{5}}{5} \vec{j} \right)$$