Tangent Vectors and Normal Vectors

Definitions of Unit Tangent Vector

Let C be a smooth curve represented by **r** on an open interval I. The unit tangent vector **T**(t) at t is defined as

$$\boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{\|\boldsymbol{r}'(t)\|}, \qquad \boldsymbol{r}'(t) \neq \mathbf{0}$$

1. Find the unit tangent vector to the curve at the specified value of the parameter
(Similar to p.865 #5-10)

$$\boldsymbol{r}(t) = 5t\mathbf{i} + t^3\mathbf{j}, \mathrm{t} = 1$$

2. Find the unit tangent vector to the curve at the specified value of the parameter
(Similar to p.865 #5-10)

$$\boldsymbol{r}(t) = 2\cos(\mathrm{t})\mathbf{i} + 3\sin(\mathrm{t})\mathbf{j}, \mathrm{t} = \frac{\pi}{6}$$

Note

Given a tangent vector T(t) at a point: (x_1, y_1, z_1):

Form: $k\langle a_1, b_1, c_1\rangle$

Direction numbers: $a = a_1$, $b = b_1$, $c = c_1$

Parametric Equations:

$x = at + x_1$, $y = bt + y_1$, $z = ct + z_1$

3. Find the unit tangent vector **T**(t) and find a set of parametric equations for the line tangent to the space curve at point P
(Similar to p.865 #11-16)

$$\boldsymbol{r}(t) = t^2\mathbf{i} + 5\mathrm{t}\mathbf{j} + \mathrm{t}\mathbf{k},\ \mathrm{P}(0, 0, 0)$$

4. Find the unit tangent vector **T**(t) and find a set of parametric equations for the line tangent to the space curve at point P
(Similar to p.865 #11-16)

$$\boldsymbol{r}(t) = <2\cos(t), 2\sin(t), 3>,\ P(\sqrt{3}, 1, 3)$$

5. Find the unit tangent vector **T**(t) and find a set of parametric equations for the line tangent to the space curve at point P
(Similar to p.865 #11-16)

$$\boldsymbol{r}(t) = < \mathrm{t}, \mathrm{t}^2, 3\mathrm{t} - 1 >,\ P(2,4,5)$$

Definition: Principal Unit Normal Vector

Let C be a smooth curve represented by **r** on an open interval I. If $\mathbf{T}'(t) \neq \mathbf{0}$, then the principal unit normal vector at t defined as

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$$

6. Find the principal unit normal vector to the curve at the specified value of the parameter
(Similar to p.866 #23-30)

$$\boldsymbol{r}(t) = \mathrm{t}^2\mathbf{i} + 5\mathrm{t}\mathbf{j},\ \mathrm{t} = 3$$

7. Find the principal unit normal vector to the curve at the specified value of the parameter
(Similar to p.866 #23-30)

$$\boldsymbol{r}(t) = ln(t)\mathbf{i} + \mathrm{t}^2\mathbf{j} + 5\mathbf{k},\ \mathrm{t} = 1$$

8. Find v(t), a(t), T(t), and N(t) (if it exists) for the an object moving along the path given by the vector-valued function r(t)
(Similar to p.866 #31-34)

$$\boldsymbol{r}(t) = 5t^2\mathbf{i} - 3\mathrm{t}\mathbf{j},\ \mathrm{t} = 1$$

Tangential and Normal Components of Acceleration

If **r**(t) is the position vector for a smooth curve C [for which **N**(t) exists], then the tangential and normal components of acceleration are as follows:

$$a_{\mathbf{T}} = D_t[\|\mathbf{v}\|] = \mathbf{a}\cdot\mathbf{T} = \frac{\mathbf{v}\cdot\mathbf{a}}{\|\mathbf{v}\|}$$

$$a_{\mathbf{N}} = \|\mathbf{v}\|\|\mathbf{T}'\| = \mathbf{a}\cdot\mathbf{N} = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|} = \sqrt{\|\mathbf{a}\|^2 - \mathbf{a}_{\mathbf{T}}^2}$$

9. Find T(t), N(t), a_T, and a_N at the given time t for the plane curve r(t) (Similar to p.866 #35-44)

$$\boldsymbol{r}(t) = (1 - t^2)\mathbf{i} + 2\text{t}^2\mathbf{j}, \text{t} = 2$$

10. Find T(t), N(t), a_T, and a_N at the given time t for the plane curve r(t) (Similar to p.866 #35-44)

$$\boldsymbol{r}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j}, \text{t} = 0$$

11. Sketch the graph of the plane curve given by the vector-valued function, and, at the point on the curve determined by r(t_o), sketch the vectors T and N. Note that N points toward the concave side of the curve (Similar to p.866 #49-54)

$$\boldsymbol{r}(t) = 5\text{t}^2\mathbf{i} + 5t\mathbf{j}, \text{t}_0 = \frac{1}{5}$$

12. Sketch the graph of the plane curve given by the vector-valued function, and, at the point on the curve determined by r(t_o), sketch the vectors T and N. Note that N points toward the concave side of the curve (Similar to p.866 #49-54)

$$\boldsymbol{r}(t) = 3\cos(\text{t})\ \mathbf{i} + 3\sin(t)\mathbf{j}, \text{t}_0 = \frac{\pi}{2}$$

13. Find T(t), N(t), a_T, and a_N at the given time t for the space curve r(t) [Hint: Find a(t), T(t), a_T, and a_N. Solve for N in the equation a(t) = a_TT + a_NN.] (Similar to p.866 #55-62)

$$\boldsymbol{r}(t) = 5\text{t}\ \mathbf{i} + 4t\mathbf{j}\ - 2\text{t}\mathbf{k}, \text{t} = 2$$