Triple Integrals and Applications

1. Evaluate the iterated integral
(Similar to p.1035 #1-8)

$$\int_0^2 \int_0^3 \int_0^1 (2x - y + z)\, \mathrm{dx\ dy\ dz}$$

2. Evaluate the iterated integral
(Similar to p.1035 #1-8)

$$\int_0^1 \int_0^2 \int_0^x (ze^{x^2})\, \mathrm{dy\ dx\ dz}$$

Volume

If f is continuous over a bounded solid region Q, then the volume of the solid region Q is given by:

$$V = \iiint_Q dV$$

3. Set up a triple integral for the volume of the solid (Hint: Plug zeros in for the variable(s) that are gone and solve for the current variable)
(Similar to p.1035 #13-18)

The solid in the first octant bounded by the coordinate planes and the plane $z = 2 - x - y$

4. Set up a triple integral for the volume of the solid (Hint: Plug zeros in for the variable(s) that are gone and solve for the current variable)
(Similar to p.1035 #13-18)

The solid bounded by $z = 4 - x^2 - y^2$ and $z = 0$

5. Use a triple integral to find the volume of the solid bounded by the graphs of the equations (Similar to p.1035 #23-26)

$z = 9 - x^2,\ y = 9 - x^2,$ first octant

6. Use a triple integral to find the volume of the solid bounded by the graphs of the equations (Similar to p.1035 #23-26)

$z = 3 - y, z = 9 - y^2, x = 0, x = 5, y = 0$

7. Sketch the solid whose volume is given by the iterated integral and rewrite the integral using the indicated order of integration (Similar to p.1035 #27-32)

$$\int_0^3 \int_{-2}^0 \int_0^{y^2} dz\, dy\, dx$$

rewrite using the order dy dz dx

$m = \iiint_Q \rho(x, y, z)dV$ Mass of the solid

$M_{yz} = \iiint_Q x\rho(x, y, z)dV$ First moment about yz - plane

$M_{xz} = \iiint_Q y\rho(x, y, z)dV$ First moment about xz - plane

$M_{xy} = \iiint_Q z\rho(x, y, z)dV$ First moment about xy - plane

$\bar{x} = \frac{M_{yz}}{m} \quad \bar{y} = \frac{M_{xz}}{m} \quad \bar{z} = \frac{M_{xy}}{m}$

Center of mass $= (\bar{x}, \bar{y}, \bar{z})$

8. Find the mass and the indicated coordinates of the center of mass of the solid of given density bounded by the graphs of the equations (Similar to p.1036 #39-42)

Find $\bar{x}$ using $\rho(x, y, z) = k$

$Q: 3x + 6y + 2z = 24,\ x = 0,\ y = 0,\ z = 0$

$I_x = \iiint_Q (y^2 + z^2)\rho(x, y, z)dV$ Moment of inertia about x - axis

$I_y = \iiint_Q (x^2 + z^2)\rho(x, y, z)dV$ Moment of inertia about y - axis

$I_z = \iiint_Q (x^2 + y^2)\rho(x, y, z)dV$ Moment of inertia about z - axis

9. Set up a triple integral that gives the moments of inertia
(Similar to p.1037 #61-62)

$$Q = \{(x, y, z) : -2 \le x \le 2, -3 \le y \le 3, 0 \le z \le 3 - x\}$$
$$\rho = \sqrt{x^2 + y^2 - z^2}$$

10. Using the description of the solid region, set up the integral for (a) the mass, (b) the center of mass, and (c) the moment of inertia about the z-axis
(Similar to p.1037 #63-64)

The solid bounded by $z = 9 - x^2 - y^2$ and $z = 0$ with density function $\rho = kz$