7. $\vec{F}(x,y) = \underbrace{2y}_{M}\,\vec{i} + \underbrace{2x}_{N}\,\vec{j}$

$$\frac{\partial M}{\partial y} \stackrel{?}{=} \frac{\partial N}{\partial x}$$

$2 = 2$ ✓

$f_x = 2y$ $\qquad$ $f_y = 2x$

$f(x,y) = \int (2y)\,dx$ $\qquad$ $f(x,y) = \int (2x)\,dy$

$= 2xy + K$ $\qquad$ $= 2xy + K$

so

$f(x,y) = 2xy + K$

8. $\vec{F}(x,y) = \underbrace{3x^2y}_{M}\,\vec{i} + \underbrace{x^3}_{N}\,\vec{j}$

"$f_x$" $\qquad$ "$f_y$"

$$\frac{\partial M}{\partial y} \stackrel{?}{=} \frac{\partial N}{\partial x}$$

$3x^2 = 3x^2$ ✓

$f_x = 3x^2y$ $\qquad$ $f_y = x^3$

$f(x,y) = \int 3x^2y\,dx$ $\qquad$ $f(x,y) = \int x^3\,dy$

$= 3\cdot\frac{x^3}{3}y + K_1$ $\qquad$ $= x^3y + K$

$= x^3y + K$

so

$f(x,y) = x^3y + K$

9. $\vec{F}(x,y) = \underbrace{3e^xy}_{M}\,\vec{i} + \underbrace{x^3}_{N}\,\vec{j}$

$$\frac{\partial M}{\partial y} \stackrel{?}{=} \frac{\partial N}{\partial x}$$

$3e^x \stackrel{?}{=} 3x^2$

NOT CONSERVATIVE