10. $\vec{F}(x,y,z) = \underbrace{x^2 y}_{M}\,\vec{i} \underbrace{-3xyz}_{N}\,\vec{j} + \underbrace{4xz}_{P}\,\vec{k}$

$$\operatorname{curl} \vec{F}(x,y,z) = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\vec{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\vec{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\vec{k}$$

$$= \left(0 - (-3xy)\right)\vec{i} - (4z - 0)\,\vec{j} + (-3yz - x^2)\,\vec{k}$$

$$= 3xy\,\vec{i} - 4z\,\vec{j} + (-3yz - x^2)\,\vec{k}$$

11. $\vec{F}(x,y,z) = \underbrace{x^2 y^3 z^3}_{M}\,\vec{i} + \underbrace{x^3 y^2 z^3}_{N}\,\vec{j} + \underbrace{x^3 y^3 z^2}_{P}\,\vec{k}$

$\frac{\partial P}{\partial y} \overset{?}{=} \frac{\partial N}{\partial z}$ $\qquad$ $\frac{\partial P}{\partial x} \overset{?}{=} \frac{\partial M}{\partial z}$ $\qquad$ $\frac{\partial N}{\partial x} \overset{?}{=} \frac{\partial M}{\partial y}$

$3x^3y^2z^2 \overset{?}{=} 3x^3y^2z^2$ $\qquad$ $3x^2y^3z^2 \overset{?}{=} 3x^2y^3z^2$ $\qquad$ $3x^2y^2z^3 \overset{?}{=} 3x^2y^2z^3$

✓ $\qquad$ ✓ $\qquad$ ✓

YES, CONSERVATIVE

$f_x = x^2y^3z^3$ $\qquad$ $f_y = x^3y^2z^3$ $\qquad$ $f_z = x^3y^3z^2$

$f(x,y,z) = \int (x^2y^3z^3)\,dx$ $\qquad$ $f(x,y,z) = \int (x^3y^2z^3)\,dy$ $\qquad$ $f(x,y,z) = \int (x^3y^3z^2)\,dz$

$= \frac{1}{3}x^3y^3z^3 + g(y,z)$ $\qquad$ $= \frac{1}{3}x^3y^3z^3 + h(x,z)$ $\qquad$ $= \frac{1}{3}x^3y^3z^3 + \ell(x,y)$

$$f(x,y,z) = \frac{1}{3}x^3y^3z^3 + K$$