Vector Fields

1. Computer ||F|| and sketch several representative vectors in the vector field
(Similar to p.1067 #7-16)

$$\mathbf{F}(x, y) = y\mathbf{i} + 2x\mathbf{j}$$

2. Find the conservative vector field for the potential function by finding its gradient
(Similar to p.1067 #21-29)
Hint: F(x, y) = f_x**i** + f_y**j**

$$f(x, y) = x^3 + 4y^2$$

3. Find the conservative vector field for the potential function by finding its gradient
(Similar to p.1067 #21-29)
Hint: F(x, y) = f_x**i** + f_y**j**

$$f(x, y, z) = e^{3x} - e^{y^2 z^3}$$

4. Verify that the vector field is conservative
(Similar to p.1067 #31-34)
Hint: **F**(x, y) = M**i** + N**j**
conservative if:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\mathbf{F}(x, y) = x^2 y\mathbf{i} + \frac{1}{3}x^3\mathbf{j}$$

5. Determine whether the vector field is conservative.
(Similar to p.1067 #35-38)
Hint: **F**(x, y) = M**i** + N**j**
conservative if:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\mathbf{F}(x, y) = 7xy\mathbf{i} + \frac{7}{2}x^2\mathbf{j}$$

6. Determine whether the vector field is conservative.
(Similar to p.1067 #35-38)
Hint: **F**(x, y) = M**i** + N**j**
conservative if:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\mathbf{F}(x, y) = 2x^2y\mathbf{i} + x^2y^3\mathbf{j}$$

7. Determine whether the vector field is conservative. If it is, find a potential function for the vector field
(Similar to p.1067 #39-48)
Hint: **F**(x, y) = f_x**i** + f_y**j**
after you get f_x and f_y, take the integral with respect to the variable to get the potential function

$$\mathbf{F}(x, y) = 2y\mathbf{i} + 2x\mathbf{j}$$

8. Determine whether the vector field is conservative. If it is, find a potential function for the vector field
(Similar to p.1067 #39-48)
Hint: **F**(x, y) = f_x**i** + f_y**j**
after you get f_x and f_y, take the integral with respect to the variable to get the potential function

$$\mathbf{F}(x, y) = 3x^2y\mathbf{i} + x^3\mathbf{j}$$

9. Determine whether the vector field is conservative. If it is, find a potential function for the vector field
(Similar to p.1067 #39-48)
Hint: **F**(x, y) = f_x**i** + f_y**j**
after you get f_x and f_y, take the integral with respect to the variable to get the potential function

$$\mathbf{F}(x, y) = 3e^xy\mathbf{i} + x^3\mathbf{j}$$

10. Find curl F for the vector field at the given point
(Similar to p.1067 #49-52)

$$\mathbf{F}(x, y, z) = x^2\mathbf{yi} - 3xyz\mathbf{j} + 4\text{xz}\mathbf{k}$$

$$\text{curl F(x, y, z)} = \nabla \times \text{F(x, y, z)}$$

$$\text{curl F(x, y, z)} = \begin{vmatrix} \text{i} & \text{j} & \text{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$\text{curl F(x, y, z)} = \left(\frac{\partial \text{P}}{\partial \text{y}} - \frac{\partial N}{\partial z}\right)i - \left(\frac{\partial \text{P}}{\partial x} - \frac{\partial M}{\partial z}\right)j + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)k$$

Test for Conservative Vector Field in Space

Suppose that M, N, and P have continuous first partial derivatives in an open sphere Q in space. The vector field given by F(x, y, z) = Mi + Nj + Pk is conservative if and only if

Curl F(x, y, z) = 0

That is, F is conservative if and only if:

$$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}, \frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}, \text{ and } \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

11. Determine whether the vector field is conservative. If it is, find a potential function for the vector field
(Similar to p.1068 #57-62)
Hint: **F**(x, y) = f_x**i** + f_y**j** + f_z**k**
after you get f_x, f_y, and f_z, take the integral with respect to the variable to get the potential function

$$\mathbf{F}(x, y, z) = x^2y^3z^3\mathbf{i} + x^3y^2z^3\mathbf{j} + x^3y^3z^2\mathbf{k}$$

Definition of Divergence of a Vector Field

The divergence of **F**(x, y) = M**i** + N**j** is:

$$\text{div F(x, y)} = \nabla \cdot F(x, y) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

The divergence of **F**(x, y, z) = M**i** + N**j** + P**k** is:

$$\text{div F(x, y, z)} = \nabla \cdot F(x, y, z) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

If div **F** = 0, then **F** is said to be divergence free

12. Find the divergence of the vector field **F**
(Similar to p.1068 #63-66)

$$\mathbf{F}(x, y) = x^3\mathbf{i} + 4e^y\mathbf{j}$$

13. Find the divergence of the vector field **F**
(Similar to p.1068 #63-66)

$$\mathbf{F}(x, y, z) = \ln(x^2 + y)\mathbf{i} + x^2y^3\mathbf{j} + \ln(2y^2 - z^2)\mathbf{k}$$

14. Find the divergence of the vector field **F** at the given point
(Similar to p.1068 #63-66)

$$\mathbf{F}(x, y, z) = x^2yz^3\mathbf{i} + (x^2 - y^2)\mathbf{j} + z^4\mathbf{k},\ (3, 1, 1)$$