Vector-Valued Functions

Step by Step: Finding Domain of a Vector-Valued Function

1. Find the domain of each component function
2. The domain of the vector-valued function is the intersection of all the domains from step 1

Review: Finding the domain

$$f(x) = \frac{2}{x-7} \qquad g(x) = \sqrt{4x-3}$$

$$h(x) = \sqrt{x^2 - 3x} \qquad i(x) = \ln(x-5)$$

$$j(x) = \tan x, \quad 0 \le x < 2\pi$$

1. Find the domain of the vector-valued function (Similar to p.839 #1-8)

$$\mathbf{r}(t) = \frac{1}{t^2-4}\mathbf{i} + \frac{t}{3}\mathbf{j} + t\mathbf{k}$$

2. Find the domain of the vector-valued function (Similar to p.839 #1-8)

$$\mathbf{r}(t) = \mathbf{F}(t) \times \mathbf{G}(t)$$
$$where$$
$$\mathbf{F}(t) = \ln(t-2)\,\mathbf{i} + \mathrm{t}\mathbf{j} - 6t\mathbf{k}$$
$$\mathbf{G}(t) = \sqrt{t+7}\mathbf{i} - t\mathbf{k}$$

3. Evaluate (if possible) the vector-valued function at each given value of t (Similar to p.839 #9-12)

$$\mathbf{r}(t) = t^2\mathbf{i} + 4t\mathbf{j}$$

(a) r(1)
(b) r(s + 3)
(c) r(1 + Δt) – r(1)

4. Evaluate (if possible) the vector-valued function at each given value of t
(Similar to p.839 #9-12)

$$\mathbf{r}(t) = 3t^2\mathbf{i} + \ln(t)\mathbf{j} - 4\mathbf{k}$$

(a) r(4)
(b) r(-2)

5. Find $\|r(t)\|$
(Similar to p.839 #13-14)

$$\mathbf{r}(t) = 3\sqrt{t}\mathbf{i} + t\mathbf{j} - e^t\mathbf{k}$$

Review

Given: initial point P(x_1, y_1,z_1) and terminal point Q (x_2, y_2,z_2)

Direction Vector

v = <x_2-x_1, y_2-y_1, z_2-z_1> = <a, b, c>

Vector-Valued Function

r(t) = (x_1 + at)**i** + (y_1 + bt)**j** + (z_1 + ct)**k**

Parametric Equation

$x = x_1 + at$ $\quad$ $y = y_1 + bt$ $\quad$ $z = z_1 + ct$

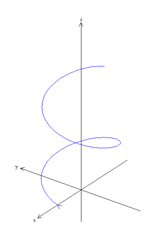
6. Represent the line segment from P to Q by a vector-valued function and by a set of parametric equations
(Similar to p.839 #15-18)

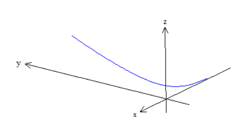
P(2, 1, -3) $\quad$ Q(4, 7, -5)

7. Find **r**(t) · **u**(t)
(Similar to p.839 #19-20)

$$\mathbf{r}(t) = (5t)\mathbf{i} + \frac{t}{4}\mathbf{j} - 5\mathbf{k}$$
$$\mathbf{u}(t) = (9t^3)\mathbf{i} + t\mathbf{j} - 7t\mathbf{k}$$

8. Match the equation with its graph.
(Similar to p.839 #21-24)





$$\mathbf{r}(t) = cos(3t)\mathbf{i} + \sin(3t)\,\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le \pi$$
$$\mathbf{r}(t) = ln(t)\mathbf{i} + 2t\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 3$$

9. Sketch the curve represented by the vector-valued function and give the orientation of the curve
(Similar to p.840 #27-42)

$$\mathbf{r}(t) = t^2\mathbf{i} + (t+3)\,\mathbf{j}$$

10. Sketch the curve represented by the vector-valued function and give the orientation of the curve
(Similar to p.840 #27-42)

$$\mathbf{r}(\theta) = (2\cos\theta)\mathbf{i} + (5\sin\theta)\mathbf{j}$$

11. Sketch the curve represented by the vector-valued function and give the orientation of the curve
(Similar to p.840 #27-42)

$$\mathbf{r}(t) = (\mathrm{t}+2)\mathbf{i} + (t-1)\mathbf{j} + (2\mathrm{t})\mathbf{k}$$

12. Sketch the curve represented by the vector-valued function and give the orientation of the curve
(Similar to p.840 #27-42)

$$\mathbf{r}(t) = t\mathbf{i} + 2\sin(\mathrm{t})\mathbf{j} + 3\cos(\mathrm{t})\mathbf{k}$$

13. Sketch the space curve represented by the intersection of the surfaces. Then represent the curve by a vector valued function using the given parameter
(Similar to p.840 #59-66)

Surfaces	Parameter
$x^2 + y^2 + z^2 = 10,$ $x + z = 4$	x = 2 + sin(t)

14. Find the limit (if it exists)
(Similar to p.840 #69-74)

$$\lim_{t\to\pi}(\sec(t)\,\mathbf{i} + \tan(t)\,\mathbf{j} - t^2\mathbf{k})$$

15. Find the limit (if it exists)
(Similar to p.840 #69-74)

$$\lim_{t \to 0} \left(\frac{\tan(t)}{t} \mathbf{i} - e^t \mathbf{j} - 3t\mathbf{k} \right)$$

16. Determine the interval(s) on which the vector-valued function is continuous
(Similar to p.841 #75-80)

$$\mathbf{r}(t) = t^2 \mathbf{i} + \frac{2}{t-3} \mathbf{j}$$