

9. $\vec{a} = \cos t \vec{i} - 2 \sin t \vec{j}$, $\vec{v}(0) = 3\vec{i} + 2\vec{j}$, $\vec{r}(0) = 2\vec{i}$

$$\vec{v}(t) = \int \vec{a} = \int (\cos t \vec{i} - 2 \sin t \vec{j}) dt$$

$$\vec{v}(t) = \sin t \vec{i} + 2 \cos t \vec{j} + \vec{C}$$

PLUG IN $t=0$

$$\vec{v}(0) = \sin 0 \vec{i} + 2 \cos 0 \vec{j} + \vec{C}$$

$$3\vec{i} + 2\vec{j} = 2\vec{j} + \vec{C}$$

$$3\vec{i} + 2\vec{j} - 2\vec{j} = \vec{C}$$

$$3\vec{i} = \vec{C}$$

so

$$\vec{v}(t) = \sin t \vec{i} + 2 \cos t \vec{j} + 3\vec{i}$$

$$\vec{v}(t) = (\sin t + 3) \vec{i} + 2 \cos t \vec{j}$$

$$\vec{r}(t) = \int \vec{v} = \int [(\sin t + 3) \vec{i} + 2 \cos t \vec{j}] dt$$

$$\vec{r}(t) = (-\cos t + 3t) \vec{i} + 2 \sin t \vec{j} + \vec{C}$$

PLUG IN $t=0$

$$\vec{r}(0) = (-\cos 0 + 3(0)) \vec{i} + 2 \sin 0 \vec{j} + \vec{C}$$

$$2\vec{i} = -\vec{i} + \vec{C}$$

$$2\vec{i} + \vec{i} = \vec{C}$$

$$3\vec{i} = \vec{C}$$

$$\vec{r}(t) = (-\cos t + 3t) \vec{i} + 2 \sin t \vec{j} + 3\vec{i}$$

$$\vec{r}(t) = (-\cos t + 3t + 3) \vec{i} + 2 \sin t \vec{j}$$

$$\text{at } t = \frac{5\pi}{3}$$

$$\vec{r}\left(\frac{5\pi}{3}\right) = \left(-\cos \frac{5\pi}{3} + 3\left(\frac{5\pi}{3}\right) + 3\right) \vec{i} + 2 \sin \frac{5\pi}{3} \vec{j}$$

$$= \left(-\frac{1}{2} + 5\pi + 3\right) \vec{i} + 2\left(-\frac{\sqrt{3}}{2}\right) \vec{j}$$

$$= \left(\frac{5}{2} + 5\pi\right) \vec{i} - \sqrt{3} \vec{j}$$