Velocity and Acceleration

Definitions of Velocity and Acceleration

If x and y are twice-differentiable functions of t, and **r** is a vector-valued function given by

r(t) = x(t)**i** + y(t)**j** (aka Position Vector), then:

Velocity = **v**(t) = **r**′(t) = x′(t)**i** + y′(t)**j**

Acceleration = **a**(t) = **r**′′(t) = x′′(t)**i** + y′′(t)**j**

$Speed = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$

1. The position vector r describes the path of an object moving in the xy-plane. Sketch a graph of the path and sketch the velocity and acceleration vectors at the given point (Similar to p.856 #1-10)

Position Function	Point
$\mathbf{r}(t) = t^2\mathbf{i} + 5t\mathbf{j}$	(1, 2)

2. The position vector r describes the path of an object moving in the xy-plane. Sketch a graph of the path and sketch the velocity and acceleration vectors at the given point (Similar to p.856 #1-10) NEXT TIME

Position Function	Point
$\mathbf{r}(t) = t^2\mathbf{i} + t^4\mathbf{j}$	(4, 16)

3. The position vector r describes the path of an object moving in the xy-plane. Sketch a graph of the path and sketch the velocity and acceleration vectors at the given point (Similar to p.856 #1-10)

Position Function	Point
$\mathbf{r}(t) = 3\cos(t)\,\mathbf{i} + 3\sin(t)\,\mathbf{j}$	(3,0)

4. The position vector r describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object (Similar to p.856 #11-20)

$$\boldsymbol{r}(t) = 4t\mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$$

5. The position vector r describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object
(Similar to p.856 #11-20)

$$\boldsymbol{r}(t) = \mathrm{t}^3\mathbf{i} + \mathrm{t}^2\mathbf{j} + 5t\mathbf{k}$$

6. The position vector r describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object
(Similar to p.856 #11-20)

$$\boldsymbol{r}(t) = <\cos(\mathrm{t}), \sin(\mathrm{t}), 5\mathrm{t}>$$

7. Use the given acceleration function to find the velocity and position vectors. Then find the position at time t = 2
(Similar to p.856 #23-28)

$$\boldsymbol{a}(t) = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$$
$$\mathbf{v}(0) = \mathbf{0}, \mathbf{r}(0) = \mathbf{0}$$

8. Use the given acceleration function to find the velocity and position vectors. Then find the position at time t = 2
(Similar to p.856 #23-28)

$$\boldsymbol{a}(t) = 2\mathrm{t}\mathbf{j} - 3\mathrm{t}\mathbf{k}$$
$$\mathbf{v}(0) = 2\mathbf{j}, \mathbf{r}(0) = 4\mathbf{k}$$

9. Use the given acceleration function to find the velocity and position vectors. Then find the position at time t = 5pi/3
(Similar to p.856 #23-28)

$$\boldsymbol{a}(t) = \cos(\mathrm{t})\,\mathbf{i} - 2\sin(\mathrm{t})\mathbf{j}$$
$$\mathbf{v}(0) = 3\mathbf{i} + 2\mathbf{j}, \mathbf{r}(0) = 2\mathbf{i}$$

Position Function for a Projectile

Neglecting air resistance, the path of a projectile launched form an initial height h with initial speed v_o, and angle of elevation θ is described by the vector function

$$\mathbf{r}(t) = (v_o \cos\theta)t\mathbf{i} + [h + (v_o \sin\theta)t - \frac{1}{2}gt^2]\mathbf{j}$$

where g is the acceleration due to gravity (32 feet per second)

10. Use the model for projectile motion, assuming there is no air resistance
(Similar to p.856 #29-44)

Find the vector-valued function for the path of a projectile launched at a height of 20 feet above the ground with an initial velocity of 100 feet per second and at an angle of 45^o above the horizontal.

11. Use the model for projectile motion, assuming there is no air resistance
(Similar to p.856 #29-44)

A baseball, hit 5 feet above the ground, leaves the bat at an angle of 30^o and is caught by an outfielder 5 feet above the ground and 200 feet from home plate. What is the initial speed of the ball, and how high does it rise?