

1. Solve:

$$-\frac{5}{6}x = 7$$

Solution (BY HAND):

$$\frac{-5}{6}x = 7$$

$$6\left(\frac{-5}{6}x\right) = 6(7)$$

$$-5x = 42$$

$$\frac{-5x}{-5} = \frac{42}{-5}$$

$$x = \frac{-42}{5}$$

1. Solve:

$$-\frac{5}{6}x = 7$$

Solution (BY TI-83):

First we want to move everything over to the left side:

$$-\frac{5}{6}x - 7 = 0$$

now we put the left side on Y1 and the right side on Y2:

“y=” button

“clear” button

“negative” button

5

“divide” button

6

“x-key” button

“minus” button

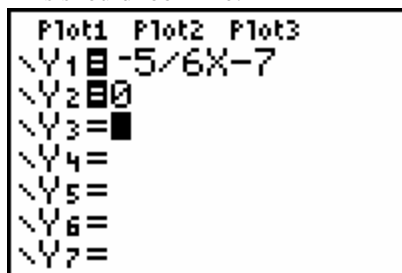
7

“down arrow” button

“clear” button

0

This should look like:



“2nd” button

“trace” button

“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

“enter” on Second Curve

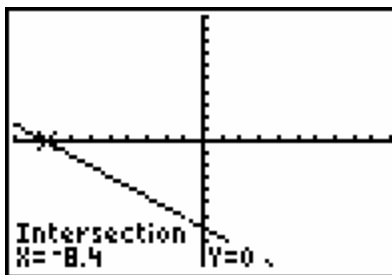
since our answer looks like it is around -9, we can enter that as the guess

1. (BY TI-83) (cont)

“negative” button

9

“enter” button



Since we came up with a decimal:

“2nd” button

“mode” button

“x-key” button

“math” button

“enter” button

“enter” button

which gives us our answer of

$$x = \frac{-42}{5}$$

2. Solve:

$$x^3 + 2x^2 - 9x - 18 = 0$$

Solution (BY HAND):

$$x^3 + 2x^2 - 9x - 18 = 0$$

$$x^2(x + 2) - 9(x + 2) = 0$$

$$(x + 2)(x^2 - 9) = 0$$

$$(x + 2)(x + 3)(x - 3) = 0$$

$$x + 2 = 0 \quad x + 3 = 0 \quad x - 3 = 0$$

$$x = -2 \quad x = -3 \quad x = 3$$

2. Solve:

$$x^3 + 2x^2 - 9x - 18 = 0$$

Solution (BY TI-83):

Since everything is already on the left side and zero on the right side, we can just plug it in:

“y=” button

“clear” button

“x-key” button

“^” button

3

“plus” button

2

“x-key” button

“x squared” button

“minus” button

9

“x-key” button

(continued on next page)

2. (BY TI-83) (cont)

“minus” button

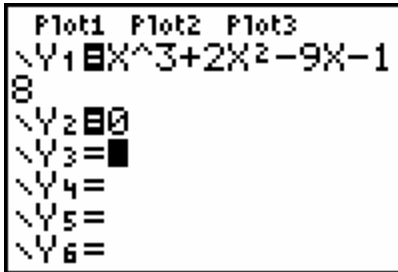
18

“down arrow” button

“clear” button

0

This should look like:



If we press “graph” button:



we see it crosses the x-axis at 3 places so we will need to do the intersect method 3 times (many times by looking at the graph, we can actually just see what the answers are, for this problem, it looks like it crosses at 3, -2, and -3 so those are probably our answers) but lets do the steps:

“2nd” button

“trace” button

“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

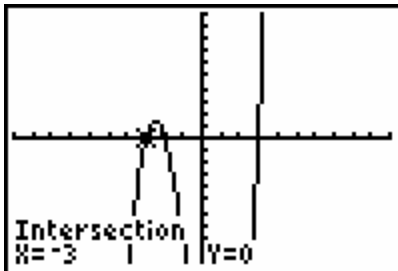
“enter” on Second Curve

Input our guess (we can guess our answers and it doesn't hurt, otherwise we just choose an x-value close to the answer):

”negative”

3

“enter”



So x = -3 is one of our answers.

Now lets do it again:

“2nd” button

“trace” button

“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

(continued in next column)

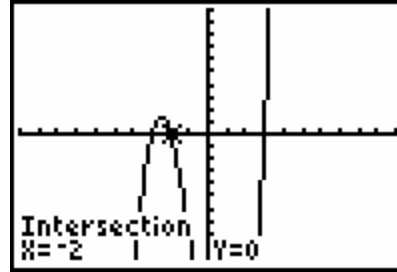
2. (BY TI-83) (cont)

“enter” on Second Curve

“negative” button

2

“enter”



so x = -2 is our second answer

now for the last one:

“2nd” button

“trace” button

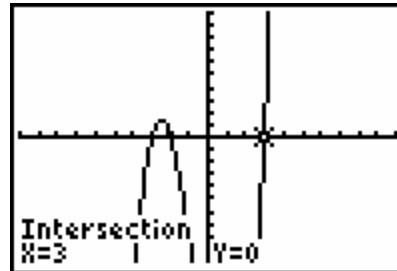
“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

“enter” on Second Curve

4

“enter”



so x = 3 is our last answer.

NOTE: I entered 4 for the guess just to demonstrate that you do not need to actually enter the answer but can enter the guess.

3. Solve:

$$\sqrt{x+9} - 3 = x$$

Solution (BY HAND):

$$\sqrt{x+9} - 3 = x$$

$$\sqrt{x+9} = x + 3$$

$$(\sqrt{x+9})^2 = (x+3)^2$$

$$x+9 = (x+3)(x+3)$$

$$x+9 = x^2 + 3x + 3x + 9$$

$$x+9 = x^2 + 6x + 9$$

$$0 = x^2 + 6x + 9 - x - 9$$

$$0 = x^2 + 5x$$

$$0 = x(x+5)$$

(continued on next page)

3. (BY HAND) (Cont)

$$0 = x(x+5)$$

$$x = 0 \quad x + 5 = 0$$

$$x = 0 \quad x = -5$$

When we check our answers though:

$$\sqrt{x+9} - 3 = x$$

check : $x = 0$

$$\sqrt{0+9} - 3 = 0$$

$$\sqrt{9} - 3 = 0$$

$$3 - 3 = 0$$

$$0 = 0$$

it works!

check : $x = -5$

$$\sqrt{x+9} - 3 = x$$

$$\sqrt{-5+9} - 3 = -5$$

$$\sqrt{4} - 3 = -5$$

$$2 - 3 = -5$$

$$-1 = -5$$

it doesn't check

REMEMBER: Whenever we take both sides to a power, we have to check our answers. It will give us the right ones but occasionally will give us wrong ones.

3. Solve:

$$\sqrt{x+9} - 3 = x$$

Solution (TI-83):

We need to get everything on the left side:

$$\sqrt{x+9} - 3 = x$$

$$\sqrt{x+9} - 3 - x = 0$$

then enter it in:

“y=” button

“clear” button

“2nd” button

“x squared” button

“x-key” button

“plus” button

9

“)” button

“minus” button

3

“minus” button

“x-key” button

“down arrow” button

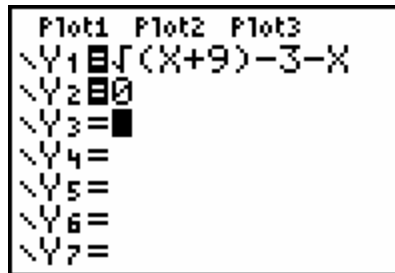
(continued in next column)

3. (BY TI-83) (Continued)

“clear” button

0

This should look like:



then:

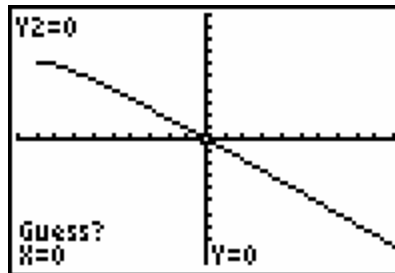
“2nd” button

“trace” button

“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

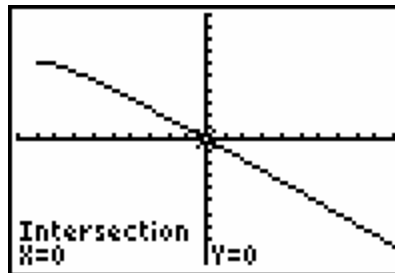
“enter” on Second Curve



It looks like our answer is at $x = 0$. We can enter 0 in as a guess but any value entered in will give us the correct answer:

2

“enter”



So our answer is $x = 0$

4. Solve the equation:

$$-\frac{1}{b} - 2 = \frac{1}{3}$$

Solution: (BY HAND):

$$-\frac{1}{b} - 2 = \frac{1}{3}$$

$$3b\left(\frac{-1}{b}\right) + 3b(-2) = 3b\left(\frac{1}{3}\right)$$

$$3(-1) - 6b = b(1)$$

(continued on next page)

4. (BY HAND) (continued)

$$3(-1) - 6b = b(1)$$

$$-3 - 6b = b$$

$$-6b - b = 3$$

$$-7b = 3$$

$$\frac{-7b}{-7} = \frac{3}{-7}$$

$$b = \frac{-3}{7}$$

4. Solve the equation:

$$\frac{-1}{b} - 2 = \frac{1}{3}$$

Solution: (BY TI-83):

We need to get everything on the left side:

$$\frac{-1}{b} - 2 - \frac{1}{3} = 0$$

now we can enter it in:

“y=” button

“clear” button

“negative” button

1

“divide” button

“x-key” button

“minus” button

2

“minus” button

1

“divide” button

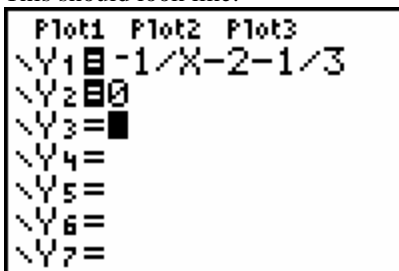
3

“down arrow” button

“clear” button

0

This should look like:



Now:

“2nd” button

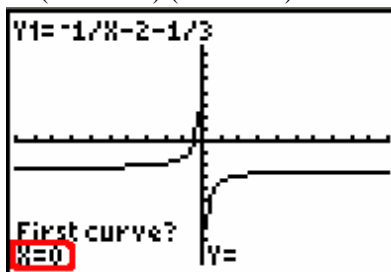
“trace” button

“down arrow” to 5:intersect and press “enter”

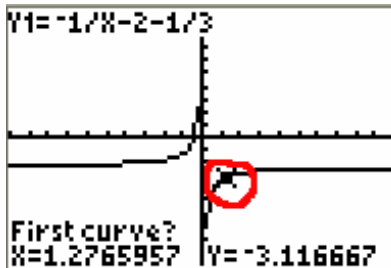
we try to “enter” on First Curve but it won’t let us.

If you look in the lower left hand corner of your calculator, it more than likely says x = 0. The graph doesn’t exist at this point

4. (BY TI-83) (continued)

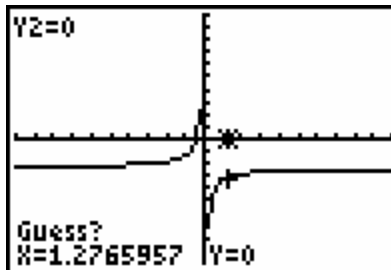


Our graph doesn’t exist at this x-value, we need to do “right arrow” until we see the cursor appear on the screen:



Now we should be able to do an “enter” on First curve

“enter” on Second curve



It looks like it might be $-1/3$ or so, lets enter -1 as the guess:

“negative” button

1

“enter” button

we get a decimal so lets try to change it to fraction:

“2nd” button

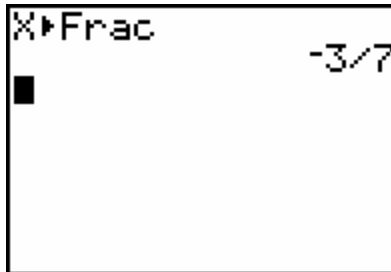
“mode” button

“x-key” button

“math” button

“enter” button

“enter” button



so our answer is $b = -3/7$

(continued in next column)

5. Solve:

$$5\sqrt{x} - 2x - 2 = 0$$

Solution (BY HAND):

$$5\sqrt{x} - 2x - 2 = 0$$

$$5\sqrt{x} = 2x + 2$$

$$(5\sqrt{x})^2 = (2x + 2)^2$$

$$5^2(\sqrt{x})^2 = (2x + 2)(2x + 2)$$

$$25x = 4x^2 + 4x + 4x + 4$$

$$25x = 4x^2 + 8x + 4$$

$$0 = 4x^2 + 8x - 25x + 4$$

$$0 = 4x^2 - 17x + 4$$

Factor using the key number method:

Key number = $ac = 4(4) = 16$

	16	
P	S	D
(1)(16)	17	15
(2)(8)	10	6
(4)(4)	8	0

we are looking for the middle number (17) which is in the first row in the S column which means our signs will be the same (both negative or both positive). In this problem they will both be negative:

$$0 = 4x^2 - 16x - 1x + 4$$

$$0 = 4x(x - 4) - 1(x - 4)$$

$$0 = (x - 4)(4x - 1)$$

$$x - 4 = 0 \quad 4x - 1 = 0$$

$$x = 4 \quad 4x = 1$$

$$x = \frac{1}{4}$$

Now lets check our answers. Remember when doing it by hand and taking both sides to a power, we have to check our answers (we may have false ones):

$$5\sqrt{x} - 2x - 2 = 0$$

$$\text{check : } x = 4$$

$$5\sqrt{4} - 2(4) - 2 = 0$$

$$5(2) - 8 - 2 = 0$$

$$10 - 8 - 2 = 0$$

$$2 - 2 = 0$$

$$0 = 0$$

(continued in next column)

5. (BY HAND) (continued)

So $x = 4$ checks

Now for the other answer:

$$5\sqrt{x} - 2x - 2 = 0$$

$$\text{check : } x = \frac{1}{4}$$

$$5\sqrt{\frac{1}{4}} - 2\left(\frac{1}{4}\right) - 2 = 0$$

$$5\left(\frac{1}{2}\right) - \frac{2}{4} - 2 = 0$$

$$\frac{5}{2} - \frac{1}{2} - 2 = 0$$

$$\frac{4}{2} - 2 = 0$$

$$2 - 2 = 0$$

$$0 = 0$$

so it checks and our answers are:

$$x = 4, x = \frac{1}{4}$$

5. Solve:

$$5\sqrt{x} - 2x - 2 = 0$$

Solution (BY TI-83)

Everything is already on the left side so we can just enter it in:

“y=” button

“clear” button

5

“2nd” button

“x squared” button

“x-key” button

“)” button

“minus” button

2

“x-key” button

“minus” button

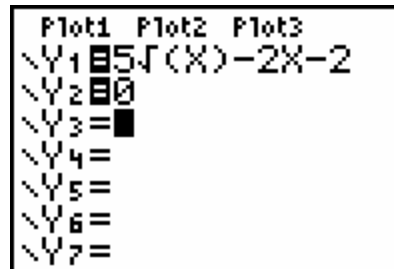
2

“down arrow” button

“clear” button

0

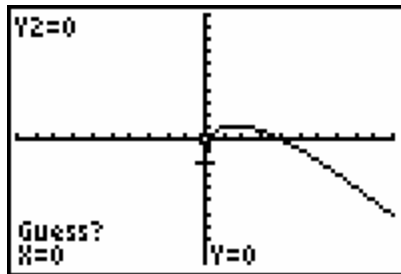
This should look like:



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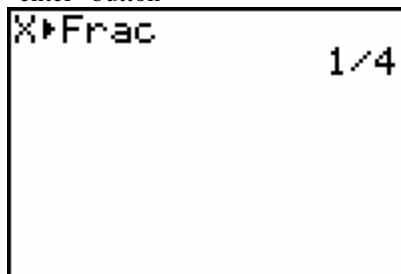
5. (BY TI-83) (continued)

Now
 “2nd” button
 “trace” button
 “down arrow” to 5:intersect and press “enter”
 “enter” on First Curve
 “enter” on Second Curve



It appears we have two answers (it crosses the x-axis in two places) so we have to do the intersect method twice. We will first guess 1 (choosing an x-value close to where the answer is:

1
 “enter” button
 we get a decimal so lets try to change it to fraction:
 “2nd” button
 “mode” button
 “x-key” button
 “math” button
 “enter” button
 “enter” button



so our first answer is

$$x = \frac{1}{4}$$

but we saw that it crossed the x-axis at another place so lets do it again:

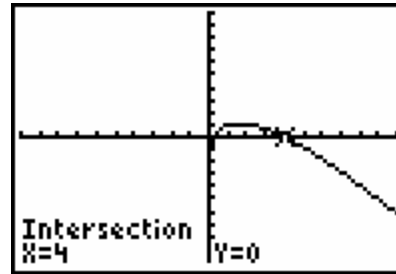
“2nd” button
 “trace” button
 “enter” on First Curve
 “enter” on Second Curve

It looks like it crosses between 3 and 5 so we can enter either one as our guess:

3
 “enter”

(continued in next column)

5. (BY TI-83) (continued)



so our second answer is $x = 4$

6. Solve:
 $-6 + n < 14$
 Solution:
 $-6 + n < 14$
 $n < 14 + 6$
 $n < 20$
 $\{n \mid n < 20\}$

7. Solve:
 $|5x + 4| < 5$
 Solution:
 $|5x + 4| < 5$
 $-5 < 5x + 4 < 5$
 $-5 - 4 < 5x < 5 - 4$
 $-9 < 5x < 1$
 $\frac{-9}{5} < \frac{5x}{5} < \frac{1}{5}$
 $\frac{-9}{5} < x < \frac{1}{5}$
 $\left\{x \mid \frac{-9}{5} < x < \frac{1}{5}\right\}$

8. Solve:
 $(x + 7)(2x - 3) \leq 0$

Solution (BY HAND):
 These are solved by the critical value method which says that we get everything on one side and zero on the other. Factor the left side. (both of these steps are done on this problem). Then find the critical values by setting each factor equal to zero and solving:

(continued on next page)

8. (BY HAND) (continued)

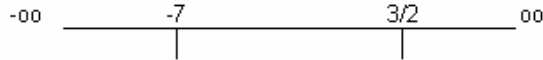
$$x + 7 = 0 \quad 2x - 3 = 0$$

$$x = -7 \quad 2x = 3$$

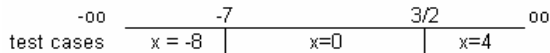
$$\frac{2x}{2} = \frac{3}{2}$$

$$x = \frac{3}{2}$$

Now we set these up in the following format:



we choose test cases that fall in the three intervals:



Now we plug each test case into our problem:

$$(x + 7)(2x - 3) \leq 0$$

and see whether it is true or false:

$$(x + 7)(2x - 3) \leq 0$$

check : $x = -8$

$$(-8 + 7)(2(-8) - 3) \leq 0$$

$$(-1)(-16 - 3) \leq 0$$

$$(-1)(-19) \leq 0$$

$$19 \leq 0$$

false

check : $x = 0$

$$(0 + 7)(2(0) - 3) \leq 0$$

$$(7)(-3) \leq 0$$

$$-21 \leq 0$$

true

check : $x = 4$

$$(4 + 7)(2(4) - 3) \leq 0$$

$$(11)(8 - 3) \leq 0$$

$$(11)(5) \leq 0$$

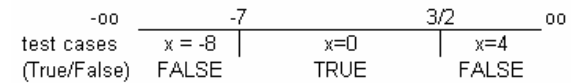
$$55 \leq 0$$

false

(continued on next column)

8. (BY HAND) (continued)

which gives us:



Our final answer is where it is true at so:

$$-7 \leq x \leq \frac{3}{2}$$

$$\left\{ x \mid -7 \leq x \leq \frac{3}{2} \right\}$$

NOTE: it is less than or equals to since our original problem had the equals to on it. And in choosing our test cases, we could have chose any value just as long as it fell in the intervals we defined. We could have chose $x = -9$ instead of $x = -8$.

8. Solve:

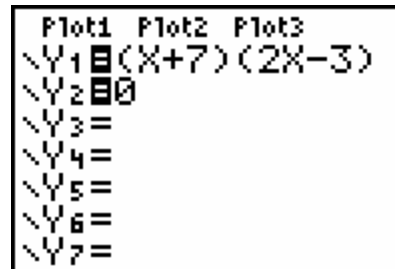
$$(x + 7)(2x - 3) \leq 0$$

Solution (BY TI-83):

Since we already have everything on the left side and zero on the right side, we can enter it in:

- “y=” button
- “clear” button
- “(“ button
- “x-key” button
- “plus” button
- 7
-)” button
- “(“ button
- 2
- “x-key” button
- “minus” button
- 3
-)” button
- “down arrow” button
- “clear” button
- 0

This should look like:

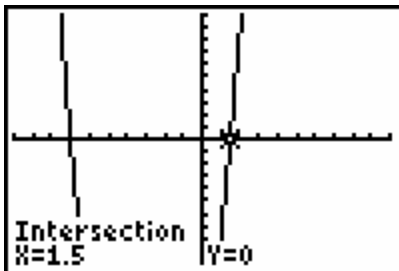


Now we use the intersect method to find the critical values:

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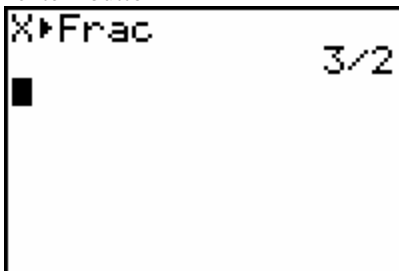
8. (BY TI-83) (continued)

“2nd” button
 “trace” button
 “down arrow” to 5:intersect and press “enter”
 “enter” on First Curve
 “enter” on Second Curve
 5
 “enter”



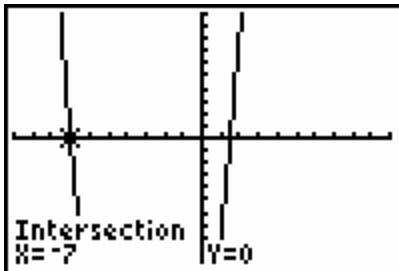
Since we have decimal, we need to change it to fraction form (if you don't recognize it) so:

“2nd” button
 “mode” button
 “x-key” button
 “math” button
 “enter” button
 “enter” button



So this is our first critical value
 NOTE: I chose 5 as the guess to demonstrate, it doesn't have to be real close to your answer. It will find the answer that is closest to your guess
 So...

“2nd” button
 “trace” button
 “down arrow” button to 5:intersect and “enter”
 “enter” on First Curve
 “enter” on Second Curve
 “negative” button
 5
 “enter”



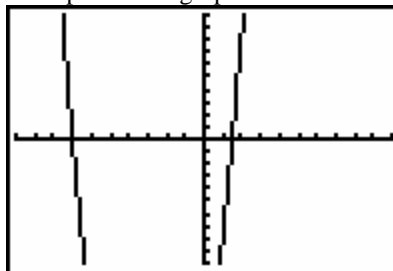
which gives us our other critical value
 (continued in next column)

8. (BY TI-83) (continued)

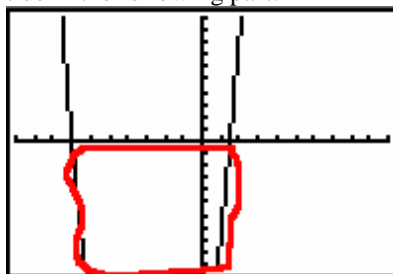
Our two critical values are:

$$x = -7, x = \frac{3}{2}$$

if we press the “graph” button:



Since our original problem was looking for where it (y values) was less than or equal to zero, we see it is true in the following part:



so that is where our answer is:

$$-7 \leq x \leq \frac{3}{2}$$

$$\left\{ x \mid -7 \leq x \leq \frac{3}{2} \right\}$$

9. Find the standard form of the equation of the specified circle. Center: (-3, 3); Radius: $6\sqrt{5}$

Solution:

From our formulas, we have:

$$(x - h)^2 + (y - k)^2 = r^2$$

center : (h, k)

radius : r

so h = -3 and k = 3 and r = $6\sqrt{5}$

this isn't so much working the problem as just plugging it into a formula:

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - (-3))^2 + (y - 3)^2 = (6\sqrt{5})^2$$

$$(x + 3)^2 + (y - 3)^2 = 6^2 (\sqrt{5})^2$$

$$(x + 3)^2 + (y - 3)^2 = 36(5)$$

$$(x + 3)^2 + (y - 3)^2 = 180$$

10. Evaluate the function and simplify the results.

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } x < -4 \\ -5 - 4x^2 & \text{if } x \geq -4 \end{cases}$$

a) $f(-4)$

b) $f(0)$

c) $f(1)$

d) $f(-3.9)$

Solution:

To evaluate these, we need to see which interval it falls into (this is the part that follows the "if" in the problem). For example, on the $\frac{1}{2}x$, this is true when $x < -4$ which are numbers like $-8, -7, -6, -5, -4.5, -4.1$.

So

a) $f(-4)$

falls in second category

$$f(-4) = -5 - 4(-4)^2$$

$$f(-4) = -5 - 4(16)$$

$$f(-4) = -5 - 64$$

$$f(-4) = -69$$

b) $f(0)$

falls in second category

$$f(0) = -5 - 4(0)^2 = -5 - 0 = -5$$

c) $f(1)$

falls in second category

$$f(1) = -5 - 4(1)^2 = -5 - 4 = -9$$

d) $f(-3.9)$

falls in second category

$$f(-3.9) = -5 - 4(-3.9)^2 = -65.84$$

11. Which shows the equation of a line, in slope-intercept form, that passes through the point (6, -4) with slope 2?

Solution:

Slope intercept form looks like: $y = mx + b$

We are given the slope so we plug that in:

$$y = 2x + b$$

and we are given a point (which corresponds to our x and y values) so we plug it in to find b:

(continued in next column)

11. (continued)

$$y = 2x + b$$

$$-4 = 2(6) + b$$

$$-4 = 12 + b$$

$$b = -16$$

so our final answer (considering $m=2$ and $b=-16$):

$$y = 2x - 16$$

12. State whether the function has a minimum or maximum value and find the value.

$$f(x) = -6x^2 - 30x$$

Solution (BY HAND):

We compare it to our quadratic form:

$$f(x) = ax^2 + bx + c$$

and find that $a = -6$ and $b = -30$

Since a is negative, we will have a maximum and we plug it into our vertex formula to find the value:

$$\text{vertex} = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$$

$$\text{vertex} = \left(\frac{-(-30)}{2(-6)}, f\left(\frac{-(-30)}{2(-6)}\right) \right)$$

$$\text{vertex} = \left(\frac{30}{-12}, f\left(\frac{30}{-12}\right) \right)$$

$$\text{vertex} = \left(\frac{5}{-2}, f\left(\frac{5}{-2}\right) \right)$$

Our max or min always is referring to our y part of the vertex so we need to find:

$$f\left(\frac{5}{-2}\right) = -6\left(\frac{5}{-2}\right)^2 - 30\left(\frac{5}{-2}\right)$$

$$= -6\left(\frac{25}{4}\right) + \frac{150}{2}$$

$$= \frac{-150}{4} + \frac{300}{4}$$

$$= \frac{150}{4}$$

$$= \frac{75}{2}$$

$$= 37.5$$

12. State whether the function has a minimum or maximum value and find the value.

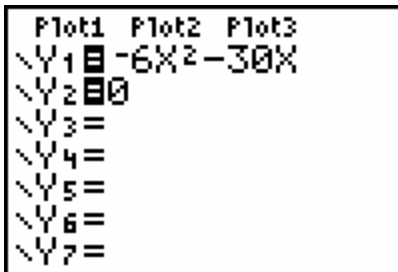
$$f(x) = -6x^2 - 30x$$

Solution (BY TI-83):

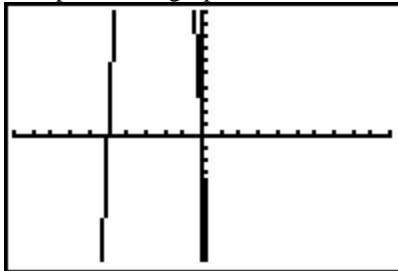
We first plug it into the calculator:

“y=” button
 “clear” button
 “negative” button
 6
 “x-key” button
 “x squared” button
 “minus” button
 30
 “x-key” button

This should look like:



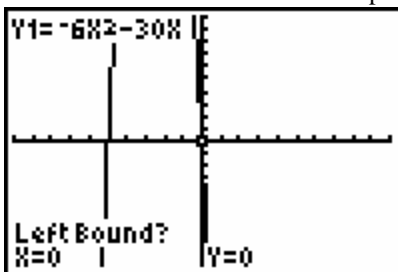
now press the “graph” button:



While we cannot see the top of the graph (we don't need to). We can tell it is a maximum that we are trying to find. It looks like it might be at $x = -3$.

Now

“2nd” button
 “trace” button
 “down arrow” to 4:maximum and press “enter”



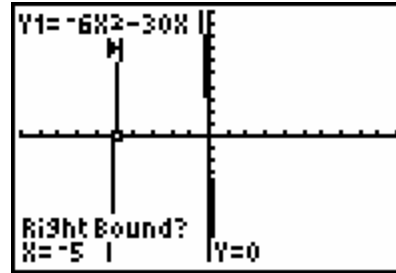
It is asking for the Left Bound?. This is an x-value that is to the left of where we think the answer is. We already said that we think that it is at $x = -3$ so let's choose an x-value definitely to the left of $x = -3$ like $x = -5$ so:

“negative” button
 5

“enter” button

(continued in next column)

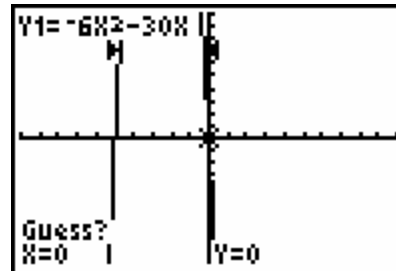
12. (BY TI-83) (continued)



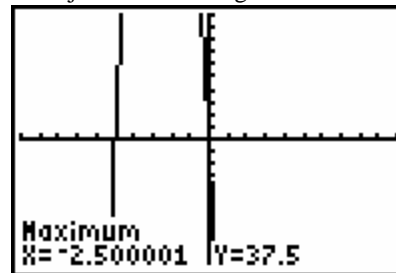
Now we need to choose a value definitely to the right of where we think the answer is like $x = 0$:

0

“enter” button



Now just “enter” on guess:

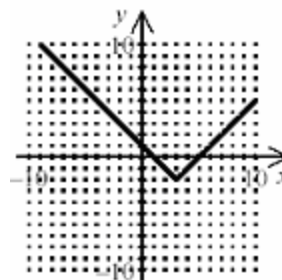


and the “y=” part is our answer so maximum at 37.5

13. Use the graph of $f(x) = |x|$ to identify the graph of $f(x) = |x - 3| - 2$

Solution (BY HAND):

You need to remember that the graph of the absolute value of x is a “v” shape starting at the origin. The -2 at the end of the new function means it moves down 2 units and the -3 inside the absolute value means it moves right 3 units (opposite of what you might think) so we are looking for a “v” shape that has been shifted down 2 units and right 3 units which matches



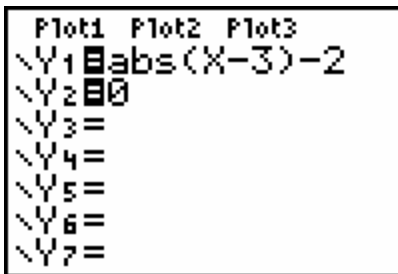
13. Use the graph of $f(x) = |x|$ to identify the graph of $f(x) = |x-3|-2$

Solution (BY TI-83):

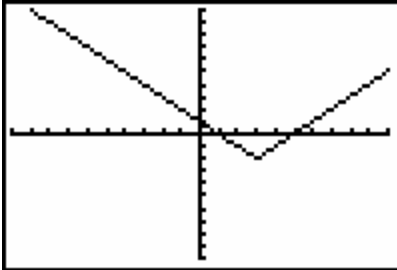
This is simple. Plug it in and see which one it matches:

“y=” button
 “clear” button
 “math” button
 “right arrow” to Num
 “enter” on abs(
 “x-key” button
 “minus” button
 3
 “)” button
 “minus” button
 2

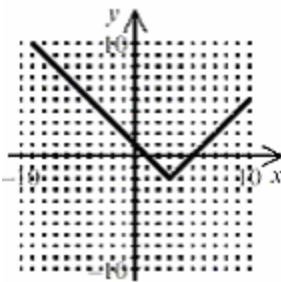
This should look like:



“graph” button



and we see it matches:



14. Given $f(x) = \frac{7}{x+4}$ and $g(x) = \frac{2}{x-4}$,

find $(f+g)(x)$

Solution:

$$\begin{aligned} (f+g)(x) &= \frac{7}{x+4} + \frac{2}{x-4} \\ &= \frac{7(x-4)}{(x+4)(x-4)} + \frac{2(x+4)}{(x+4)(x-4)} \\ &= \frac{7x-28}{(x+4)(x-4)} + \frac{2x+8}{(x+4)(x-4)} \\ &= \frac{7x-28+2x+8}{(x+4)(x-4)} \\ &= \frac{9x-20}{(x+4)(x-4)} \end{aligned}$$

15. Find $(g \circ f)(x)$ and $(f \circ g)(x)$ for

$f(x) = x+5$ and $g(x) = \sqrt{x+4}$

Solution:

$$\begin{aligned} g \circ f &= g(f) \\ &= \sqrt{(x+5)+4} \\ &= \sqrt{x+9} \end{aligned}$$

and

$$\begin{aligned} f \circ g &= f(g) \\ &= (\sqrt{x+4})+5 \\ &= \sqrt{x+4}+5 \end{aligned}$$

16. Find all relative extrema of the function:

$f(x) = 8x^3 - 3x^4$

Solution (BY TI-83):

Lets plug it in:

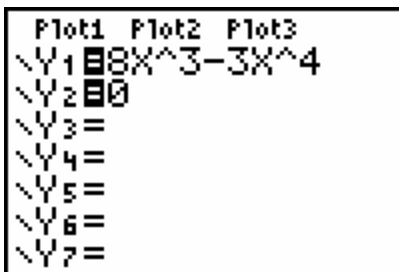
“y=” button
 “clear” button
 8
 “x-key” button
 “^” button
 3
 “minus” button

(continued on next page)

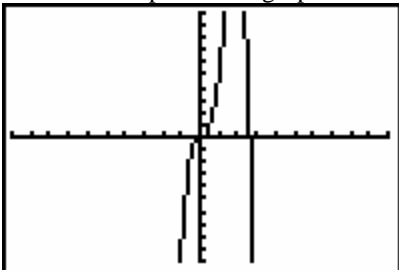
16. (BY TI-83) (continued)

3
 “x-key” button
 “^” button

4
 which should look like:

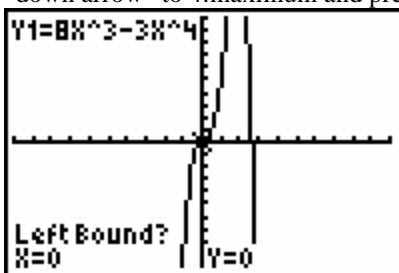


and when we press the “graph” button:



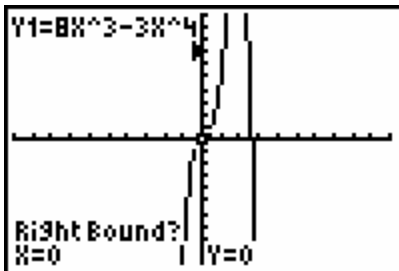
Even though we cannot see the entire graph, it obviously has a max, perhaps somewhere around $x = 2$. So...

“2nd” button
 “trace” button
 “down arrow” to 4: maximum and press “enter”



Now we need to enter an x-value to the left of where we think the answer is so lets enter $x = 0$:

0
 “enter”

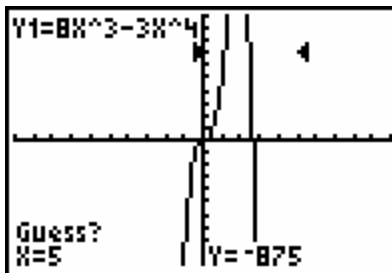


Now we need to enter an x-value to the right of where we think the answer is so lets enter $x = 5$:

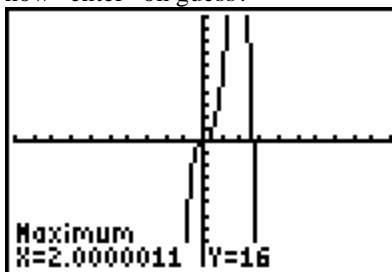
5
 “enter”

(continued in next column)

16. (BY TI-83) (continued)



now “enter” on guess?



so we have a relative maximum at (2, 16)

NOTE: the decimal places you see there are a numerical error that pops us every once in awhile. It will be quite a few decimal places out. Example: 2.5 is not a numerical error but is the answer but 2.000000011 rounds to 2.

17. Find all real zeros of the function:

$$f(x) = -4x^4 + 256x^2$$

Solution: (BY HAND):

$$-4x^4 + 256x^2 = 0$$

$$-4x^2(x^2 - 64) = 0$$

$$-4x^2(x + 8)(x - 8) = 0$$

$$-4(x)(x)(x + 8)(x - 8) = 0$$

$$x = 0 \quad x = 0 \quad x + 8 = 0 \quad x - 8 = 0$$

$$x = -8 \quad x = 8$$

$$x = 0$$

$$x = -8$$

$$x = 8$$

17. Find all real zeros of the function:

$$f(x) = -4x^4 + 256x^2$$

Solution: (BY TI-83):

“y=” button
 “clear” button
 “negative” button

4
 “x-key” button
 “^” button

4
 (continued on next page)

17. (BY TI-83) (continued)

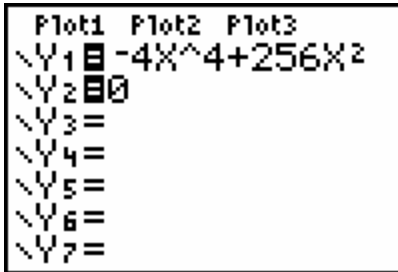
“plus” button

256

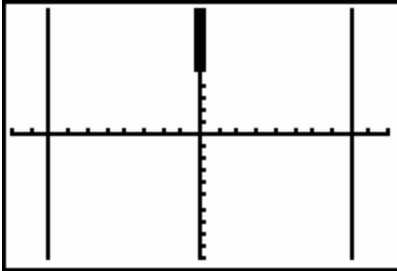
“x-key” button

“x squared” button

This should look like:



If we look at the graph (“graph” button):



It looks like it may be doing something in the middle but we will find the other two answers first

so:

“2nd” button

“trace” button

“down arrow” to 5:intersect and “enter”

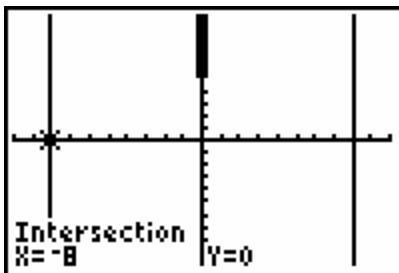
“enter” on first curve

“enter” on second curve

“negative”

6

“enter”



which gives us our first answer of $x = -8$

now

“2nd” button

“trace” button

“down arrow” to 5:intersect and “enter”

“enter” on first curve

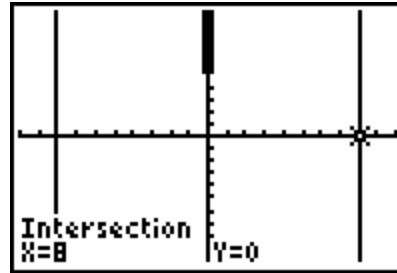
“enter” on second curve

6

“enter”

(continued in next column)

17. (BY TI-83) (continued)



which gives us our second answer of $x = 8$

Lets zoom in the middle to see what is going on:

“zoom” button

“down arrow” to 6:Zstandard and press “enter”

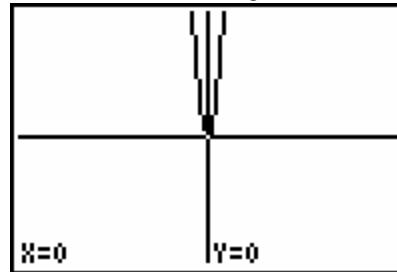
“zoom” button

“down arrow” to 2:Zoom In and press “enter”

“enter” button



Still cannot see it real good so “enter” again:



It looks like it comes down and touches at zero and goes back up so:

“2nd” button

“trace” button

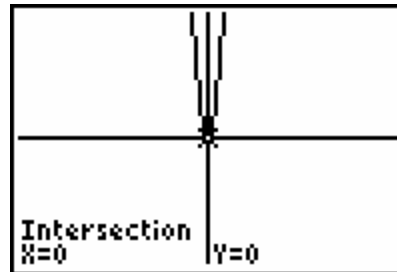
“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

“enter” on Second Curve

0

“enter” button



which gives us $x = 0$. Often times when it comes down and touches at a point, the intersect (and zero) method doesn't work unless our guess is actually the answer! Remember to Zoom-Z Standard when done

--	--

18. Find all the zeros of the function:

$$f(x) = 10x^4 - 9x^3 - 8x^2 + 9x - 2$$

Solution (BY HAND):

First lets find all of our possible rational zeros:

$$p = 2 \text{ and } q = 10$$

then:

$$p : \pm 1, \pm 2$$

$$q : \pm 1, \pm 2, \pm 5, \pm 10$$

$$\frac{p}{q} : \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{1}{10}, \pm \frac{2}{10}$$

$$\frac{p}{q} : \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{1}{10}$$

Note: there are certain rules that will help you eliminate possibilities in some problems but I find students have more success just brute force plugging in values:

	x^4	x^3	x^2	x	"no x"
1	10	-9	-8	9	-2
		10	1	-7	2
	10	1	-7	2	0

Since we got a remainder of 0, that means $x = 1$ is one of our answers (we need 4 total, it always matches our largest power). After we have found the first one, we have a new problem (which ties to our bottom line of the synthetic division):

$$10x^3 + x^2 - 7x + 2 = 0$$

Our possible rational zeros are still the same. There is nothing to say that $x = 1$ is not our second answer so lets try it again:

	x^3	x^2	x	"no x"
1	10	1	-7	2
		10	11	4
	10	11	4	6

it doesn't work so lets go onto our next one:

	x^3	x^2	x	"no x"
-1	10	1	-7	2
		-10	9	-2
	10	-9	2	0

Since we got a zero as the remainder, our next answer is $x = -1$ and we have a new problem (tying to our last line of the synthetic division...remember it is one degree lower):

$$10x^2 - 9x + 2 = 0$$

We always want to get it down to the x squared level because then we can factor or use the quadratic formula to solve it.

(continued in next column)

18. (BY HAND) (continued)

$$10x^2 - 9x + 2 = 0$$

we will use the key number method:

$$ac = 10(2) = 20$$

	20	
P	S	D
(1)(20)	21	19
(2)(10)	12	8
(4)(5)	9	1

we are looking for the number in the middle which is in the last row so we will use 4 and 5 and since it is in the S column, the signs will be the same (both negative or both positive), negative in this case:

$$10x^2 - 9x + 2 = 0$$

$$10x^2 - 5x - 4x + 2 = 0$$

$$5x(2x - 1) - 2(2x - 1) = 0$$

$$(2x - 1)(5x - 2) = 0$$

$$2x - 1 = 0 \quad 5x - 2 = 0$$

$$2x = 1 \quad 5x = 2$$

$$\frac{2x}{2} = \frac{1}{2} \quad \frac{5x}{5} = \frac{2}{5}$$

$$x = \frac{1}{2} \quad x = \frac{2}{5}$$

which gives us our last two answers.

18. Find all zeros of the functions:

$$f(x) = 10x^4 - 9x^3 - 8x^2 + 9x - 2$$

Solution (BY TI-83):

First lets enter it into the calculator:

"y=" button
 "clear" button
 10
 "x-key" button
 "x-key" button
 4
 "minus" button
 9
 "x-key" button
 "x-key" button
 3
 "minus" button
 8
 "x-key" button
 "x squared" button

(continued on next page)

18. (BY TI-83) (continued)

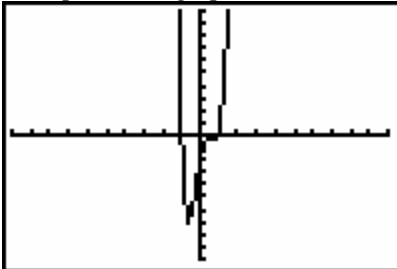
“plus” button
9
“x-key” button
“minus” button
2
“down arrow” button
0

This should look like:

```

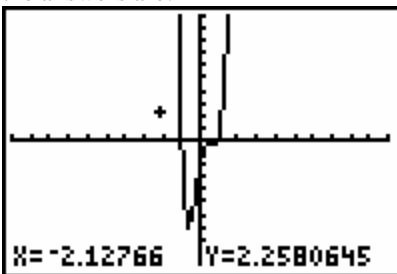
Plot1 Plot2 Plot3
\Y1=10X^4-9X^3-8
X^2+9X-2
\Y2=0
\Y3=
\Y4=
\Y5=
\Y6=
    
```

if we press the “graph” button:

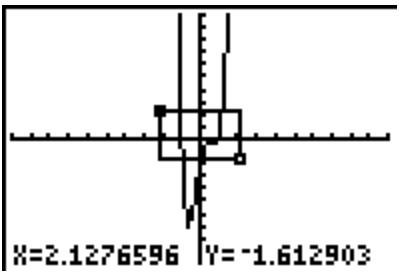


This is not so easy to see what is going on so lets zoombox:

“zoom” button
“enter” on 1:Zbox
using the “left arrow” and “up arrow” buttons, move the little plus to the left and up of where we think the answers are:

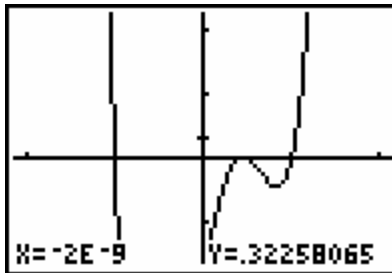


now press “enter” button
using the “right arrow” and “down arrow” buttons, move the little box to the right and down of where we think the answers are:



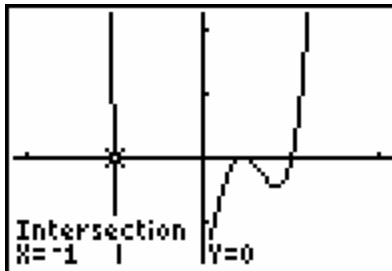
and press “enter”
(continued in next column)

18. (BY TI-83) (continued)



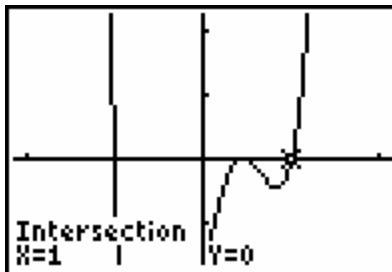
It looks like -1 and 1 are two of our answers but lets check for sure:

“2nd” button
“trace” button
“down arrow” to 5:intersect and press “enter”
“enter” on First Curve
“enter” on Second Curve
“negative” button
1
“enter”



so $x = -1$ is one of our answers
now lets check the other one:

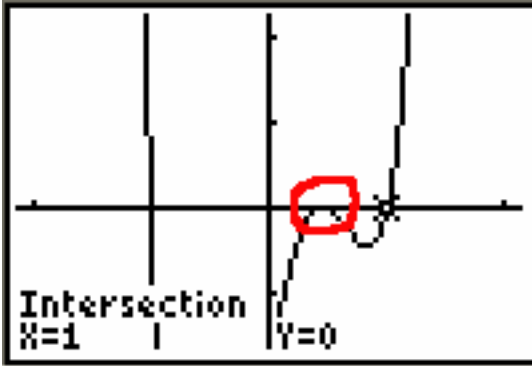
“2nd” button
“trace” button
“down arrow” to 5:intersect and press “enter”
“enter” on First Curve
“enter” on Second Curve
1
“enter”



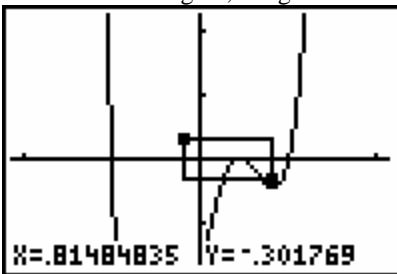
so $x = 1$ is our second answer. The question becomes what is happening at:

(continued on next page)

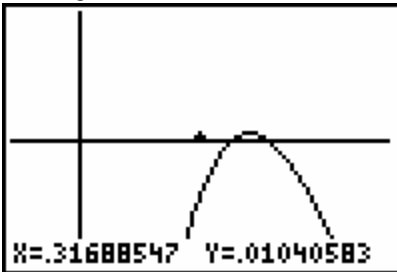
18. (BY TI-83) (continued)



when a graph comes up and touches at a point and goes back down, that answer often occurs twice but this graph doesn't look like it touches at a point and if we zoombox again, we get:



which gives us:



and we see that it crosses at two places so:

“2nd” button

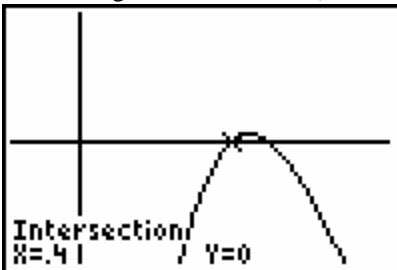
“trace” button

“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

“enter” on Second Curve

“enter” on Guess (since we cannot see our tick marks on the x-axis anymore, we will just press “enter” to get the next answer):



(continued in next column)

18. (BY TI-83) (continued)

since we have a decimal:

“2nd” button

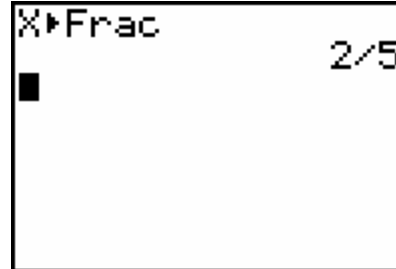
“mode” button

“x-key” button

“math” button

“enter” button

“enter” button



which gives us our third answer

so lets do it again:

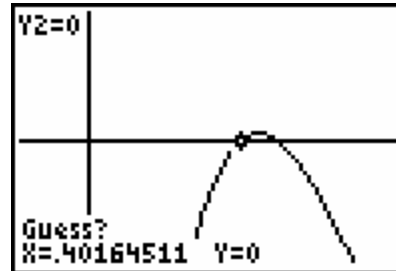
“2nd” button

“trace” button

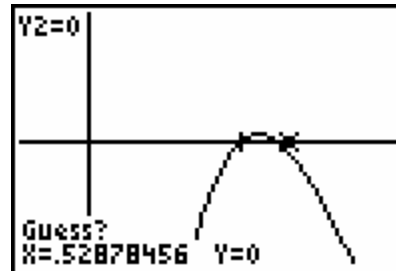
“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

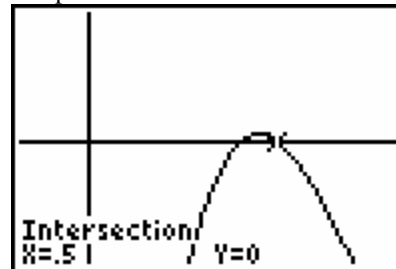
“enter” on Second Curve



If we press “enter” on guess, we will get the third answer again so we will press the “right arrow” until the cursor is closer to the other answer:



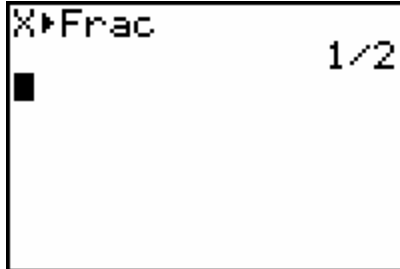
and press “enter”



(continued on next page)

18. (BY TI-83) (continued)

since we have a decimal:
 “2nd” button
 “mode” button
 “x-key” button
 “math” button
 “enter” button
 “enter” button



Which gives us our fourth answer. Now do a “zoom” button and Zstandard to set your viewing window back for future problems.

19. Find all the zeros of the function:

$$f(x) = x^4 - 6x^3 + 13x^2 + 6x - 14$$

Solution (BY HAND):

$$p = 14$$

$$q = 1$$

then:

$$p : \pm 1, \pm 2, \pm 7, \pm 14$$

$$q : \pm 1$$

$$\frac{p}{q} : \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{7}{1}, \pm \frac{14}{1}$$

$$\frac{p}{q} : \pm 1, \pm 2, \pm 7, \pm 14$$

so lets start trying our possible answers in synthetic division:

	x^4	x^3	x^2	x	$\text{"no } x\text{"}$
1	1	-6	13	6	-14
		1	-5	8	14
	1	-5	8	14	0

so $x = 1$ is our first answer and we now have the bottom row as our new problem:

$$x^3 - 5x^2 + 8x + 14 = 0$$

Lets try $x = 1$ again in the synthetic division:

	x^3	x^2	x	$\text{"no } x\text{"}$
1	1	-5	8	14
		1	-4	4
	1	-4	4	18

so it doesn't work, lets try the next possible answer.

(continued in next column)

19. (BY HAND) (continued)

	x^3	x^2	x	$\text{"no } x\text{"}$
-1	1	-5	8	14
		-1	6	-14
	1	-6	14	0

again since we got a remainder of 0, this is our second answer. And we have a new problem that ties directly to our last row:

$$x^2 - 6x + 14 = 0$$

We cannot factor this so we have to use the quadratic formula:

$$x^2 - 6x + 14 = 0$$

$$a = 1, b = -6, c = 14$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(14)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 56}}{2}$$

$$x = \frac{6 \pm \sqrt{-20}}{2}$$

$$x = \frac{6 \pm \sqrt{-(2)(2)(5)}}{2}$$

$$x = \frac{6 \pm 2\sqrt{-5}}{2}$$

$$x = \frac{6 \pm 2i\sqrt{5}}{2}$$

$$x = \frac{3 \pm 1i\sqrt{5}}{1}$$

$$x = 3 \pm i\sqrt{5}$$

--	--

19. Find all the zeros of the function:

$$f(x) = x^4 - 6x^3 + 13x^2 + 6x - 14$$

Solution (BY TI-83):

“y=” button

“clear” button

“x-key” button

“^” button

4

“minus” button

6

“x-key” button

“^” button

3

“plus” button

13

“x-key” button

“x squared” button

“plus” button

6

“x-key” button

“minus” button

14

“down arrow” button

“clear” button

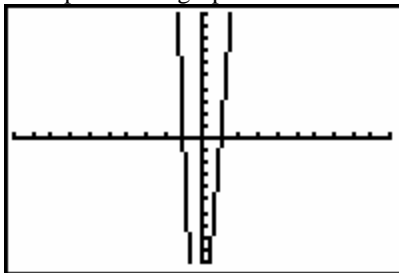
0

This should look like:

```

Plot1 Plot2 Plot3
\Y1 X^4-6X^3+13X
2+6X-14
\Y2 0
\Y3 =
\Y4 =
\Y5 =
\Y6 =
    
```

if we press the “graph” button:



It appears we have two “real” answers at $x = -1$ and $x = 1$ but lets check this just to be sure:

“2nd” button

“trace” button

“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

“enter” on Second Curve

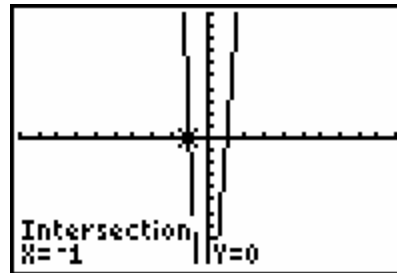
“negative” button

1

“enter”

(continued in next column)

19. (BY TI-83) (continued)



so $x = -1$ is our first answer, now lets check the other one:

“2nd” button

“trace” button

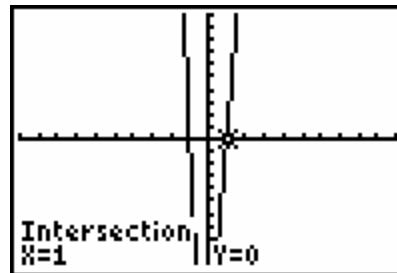
“down arrow” to 5:intersect and press “enter”

“enter” on First Curve

“enter” on Second Curve

1

“enter”



so $x = 1$ is our second answer, so now lets plug these into synthetic division:

$$\begin{array}{r|rrrrrr}
 -1 & x^4 & x^3 & x^2 & x & \text{"no x"} \\
 & 1 & -6 & 13 & 6 & -14 \\
 & & -1 & 7 & -20 & 14 \\
 \hline
 & 1 & -7 & 20 & -14 & 0
 \end{array}$$

$$\begin{array}{r|rrrr}
 1 & x^3 & x^2 & x & \text{"no x"} \\
 & 1 & -7 & 20 & -14 \\
 & & 1 & -6 & 14 \\
 \hline
 & 1 & -6 & 14 & 0
 \end{array}$$

which leaves us with:

$$x^2 - 6x + 14 = 0$$

(continued on next page)

19. (BY TI-83) (continued)

We cannot factor this so we have to use the quadratic formula:

$$x^2 - 6x + 14 = 0$$

$$a = 1, b = -6, c = 14$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(14)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 56}}{2}$$

$$x = \frac{6 \pm \sqrt{-20}}{2}$$

$$x = \frac{6 \pm \sqrt{-(2)(2)(5)}}{2}$$

$$x = \frac{6 \pm 2\sqrt{-5}}{2}$$

$$x = \frac{6 \pm 2i\sqrt{5}}{2}$$

$$x = \frac{3 \pm i\sqrt{5}}{1}$$

$$x = 3 \pm i\sqrt{5}$$

which is our last two answers

20. Find the vertical and horizontal asymptotes for the rational function:

$$f(x) = \frac{3x^2 - 3x - 1}{x^2 - x - 2}$$

Solution:

To find the vertical asymptote(s), we set the denominator equal to zero and solve:

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x - 2 = 0 \quad x + 1 = 0$$

$$x = 2 \quad x = -1$$

so we have two vertical asymptotes.

To find the horizontal asymptote, we note the degree of the top is equal to the degree of the bottom so we take the numbers off the largest powers:

(continued on next page)

20. (continued)

$$f(x) = \frac{3x^2 - 3x - 1}{x^2 - x - 2}$$

Note: there is the invisible one down below, so our horizontal asymptote is:

$$y = \frac{3}{1}$$

$$y = 3$$

While the TI-83 won't give these to us, we can use the graph to verify what we just found:

“y=” button

“clear” button

“(“ button

3

“x-key” button

“x squared” button

“minus” button

3

“x-key” button

“minus” button

1

“)” button

“divide” button

“(“ button

“x-key” button

“x squared” button

“minus” button

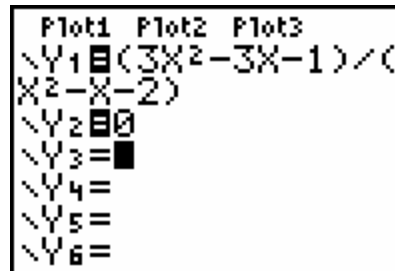
“x-key” button

“minus” button

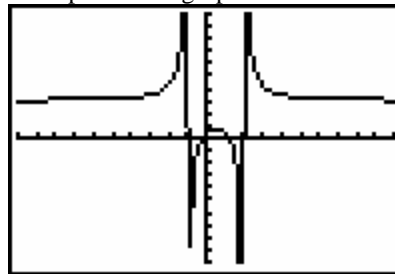
2

“)” button

This should look like:



if we press the “graph” button:



at -1 and 2, we have vertical lines (corresponds to our vertical asymptotes) and it looks like it skims the line y=3 so it checks.

21. Identify the graph of the rational function and find the equation of the slant asymptote.

$$f(x) = \frac{-2x^2 - 3x + 2}{x + 1}$$

Solution (BY HAND / TI-83):

Lets graph it:

“y=” button

“clear” button

“(“ button

“negative” button

2

“x-key” button

“x squared” button

“minus” button

3

“x-key” button

“plus” button

2

“)” button

“divide” button

“(“ button

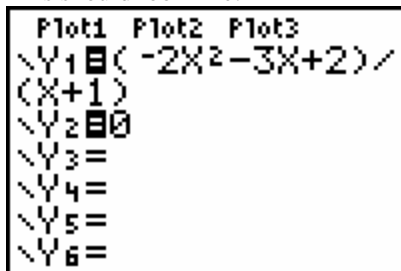
“x-key” button

“plus” button

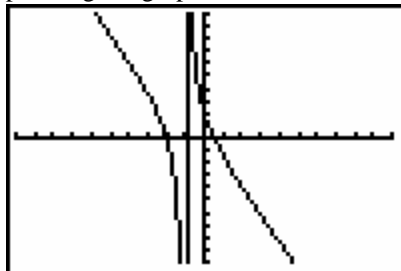
1

“)” button

This should look like:



pressing the graph button:



Our graph never includes the vertical line so cut it out.

We have a slant asymptote when the degree of the top is exactly one greater than the bottom. We have to use polynomial division and then drop off the remainder:

(continued in next column)

21. (BY HAND / TI-83) (continued)

$$\begin{array}{r} -2X - 1 \\ X + 1 \overline{) -2X^2 - 3X + 2} \\ \underline{+2X^2 + 2X} \\ -X + 2 \\ \underline{+X + 1} \\ 3 \end{array}$$

Drop off the remainder and our slant asymptote is:
 $y = -2x - 1$

22. Graph:

$$f(x) = \frac{x - 3}{x^2 + x - 12}$$

Solution (TI-83):

“y=” button

“clear” button

“(“ button

“x-key” button

“minus” button

3

“)” button

“divide” button

“(“ button

“x-key” button

“x squared” button

“plus” button

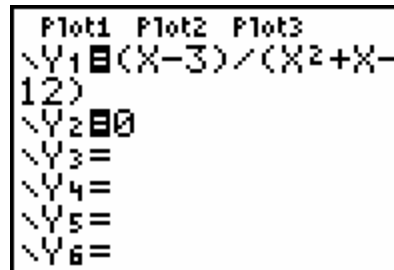
“x-key” button

“minus” button

12

“)” button

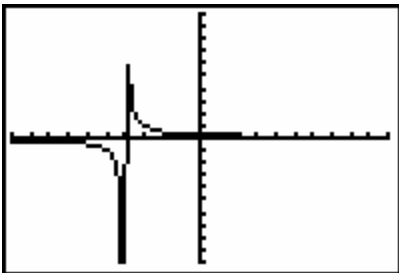
This looks like:



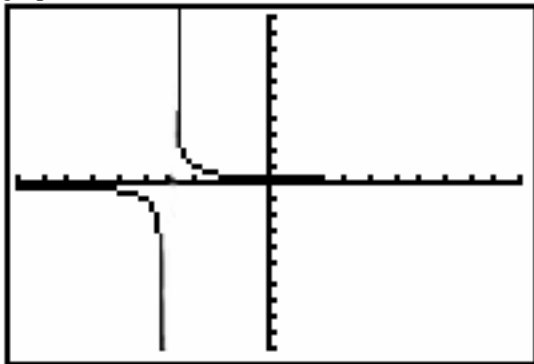
and after pressing the “graph” button

(continued on next page)

22. (BY TI-83) (continued)



Keep in mind our graph never just stops and any vertical lines are not really part of our graph so our graph looks like:



23. Find the inverse of the function:

$$f(x) = (x - 2)^3 + 4$$

Solution:

Step 1: replace f(x) with y:

$$y = (x - 2)^3 + 4$$

Step 2: interchange x and y:

$$x = (y - 2)^3 + 4$$

Step 3: solve for y:

$$x = (y - 2)^3 + 4$$

$$x - 4 = (y - 2)^3$$

$$\sqrt[3]{x - 4} = \sqrt[3]{(y - 2)^3}$$

$$\sqrt[3]{x - 4} = y - 2$$

$$2 + \sqrt[3]{x - 4} = y$$

Step 4: replace y with our inverse function notation:

$$f^{-1}(x) = 2 + \sqrt[3]{x - 4}$$

24. Identify the graph of the function:

$$f(x) = 2^x$$

Solution (TI-83):

“y=” button

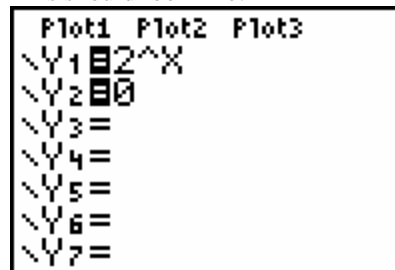
“clear” button

2

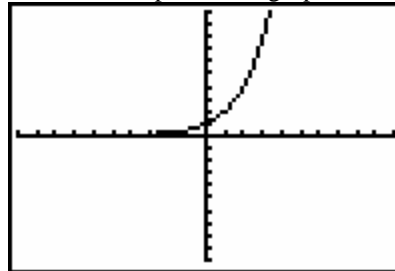
“^” button

“x-key” button

This should look like:



and when we press the “graph” button:



25. Evaluate the expression without using a calculator.

$$\log_3 \left(\frac{1}{9} \right)$$

Solution: (BY HAND):

$$\log_3 \left(\frac{1}{9} \right) = x$$

$$3^x = \frac{1}{9}$$

$$3^x = \frac{1}{3^2}$$

$$3^x = 3^{-2}$$

$$x = -2$$

Note: we set it equal to x since we are trying to find its value. The second step is using the definition of a log to rewrite it into exponential form.

25. Evaluate the expression without using a calculator.

$$\log_3\left(\frac{1}{9}\right)$$

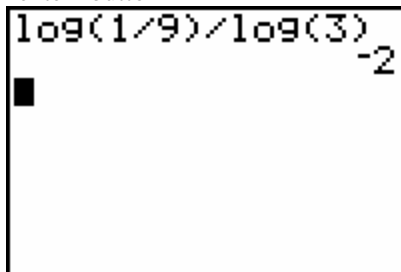
Solution: (BY TI-83):

We can use the change of base to rewrite it:

$$\log_3\left(\frac{1}{9}\right) = \frac{\log\left(\frac{1}{9}\right)}{\log(3)}$$

then:

“2nd” button
 “mode” button
 “clear” button
 “log” button
 1
 “divide” button
 9
 “)” button
 “divide” button
 “log” button
 3
 “)” button
 “enter” button



so our answer is -2

26. Solve for x:

$$\frac{1}{8} = 4^{7x-4}$$

Solution (BY HAND):

$$\frac{1}{8} = 4^{7x-4}$$

$$\frac{1}{2^3} = (2^2)^{7x-4}$$

$$2^{-3} = 2^{2(7x-4)}$$

$$2^{-3} = 2^{14x-8}$$

$$-3 = 14x - 8$$

(continued in next column)

26. (BY HAND) (continued)

$$-3 = 14x - 8$$

$$-3 + 8 = 14x$$

$$5 = 14x$$

$$\frac{5}{14} = \frac{14x}{14}$$

$$x = \frac{5}{14}$$

26. Solve:

$$\frac{1}{8} = 4^{7x-4}$$

Solution (BY TI-83):

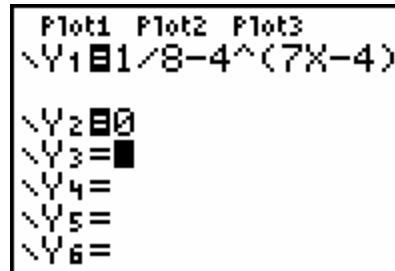
Get everything on the left side and 0 on the right side:

$$\frac{1}{8} - 4^{7x-4} = 0$$

Now

“y=” button
 “clear” button
 1
 “divide” button
 8
 “minus” button
 4
 “^” button
 “(“ button
 7
 “x-key” button
 “minus” button
 4
 “)” button
 “down arrow” button
 “clear” button
 0

This should look like:



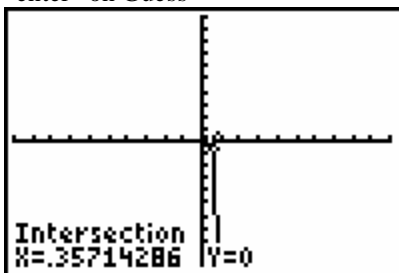
now:

“2nd” button
 “trace” button
 “down arrow” to 5:intersect and press “enter”
 “enter” on First Curve
 “enter” on Second Curve

(continued on next page)

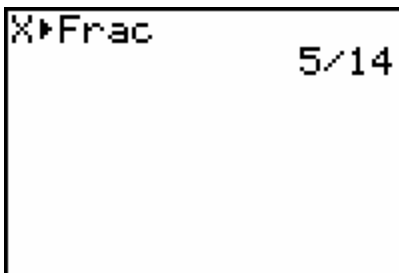
26. (BY TI-83) (continued)

“enter” on Guess



we got a decimal so:

“2nd” button
 “mode” button
 “x-key” button
 “math” button
 “enter” button
 “enter” button



which gives us our answer

27. Solve for x:

$$\left(\frac{1}{4}\right)^x = 64$$

Solution (BY HAND):

$$\left(\frac{1}{4}\right)^x = 64$$

$$\left(\frac{1}{4^1}\right)^x = 4^3$$

$$(4^{-1})^x = 4^3$$

$$4^{-1x} = 4^3$$

$$-1x = 3$$

$$\frac{-1x}{-1} = \frac{3}{-1}$$

$$x = -3$$

27. Solve for x:

$$\left(\frac{1}{4}\right)^x = 64$$

Solution (BY TI-83):

Get everything on the left side:

$$\left(\frac{1}{4}\right)^x = 64$$

$$\left(\frac{1}{4}\right)^x - 64 = 0$$

then

“y=” button
 “clear” button
 “(“ button

1
 “divide” button
 4

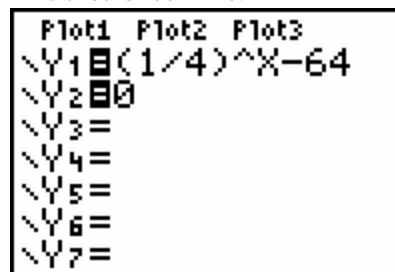
)” button
 “^” button
 “x-key” button

“minus” button
 64

“down arrow” button
 “clear” button

0

This should look like:

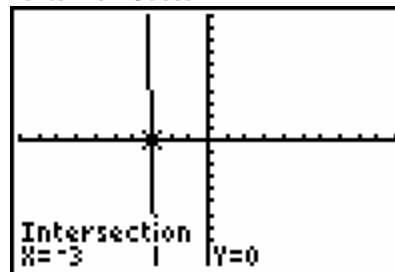


then:

“2nd” button
 “trace” button
 “down arrow” to 5:intersect and press “enter”

“enter” on First Curve
 “enter” on Second Curve

“enter” on Guess



so our answer is x = -3

28. Solve for x:

$$\ln(3x + 6) = 2$$

Solution (BY HAND):

$$\ln(3x + 6) = 2$$

$$e^2 = 3x + 6$$

$$e^2 - 6 = 3x$$

$$\frac{e^2 - 6}{3} = \frac{3x}{3}$$

$$x = \frac{e^2 - 6}{3}$$

$$x = 0.463$$

28. Solve for x:

$$\ln(3x + 6) = 2$$

Solution (BY TI-83):

We need to get everything on the left side:

$$\ln(3x + 6) = 2$$

$$\ln(3x + 6) - 2 = 0$$

then:

“y=” button

“clear” button

“ln” button

3

“x-key” button

“plus” button

6

“)” button

“minus” button

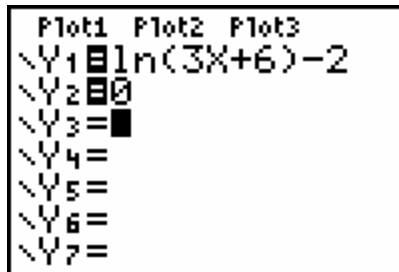
2

“down arrow” button

“clear” button

0

This should look like:



now:

“2nd” button

“trace” button

“down arrow” to 5:intersect and press “enter”

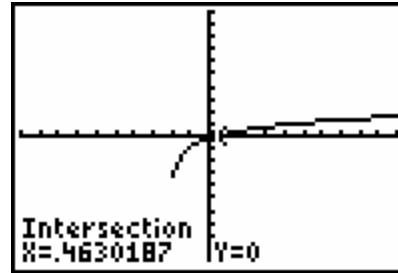
“enter” on First Curve

“enter” on Second Curve

“enter” on Guess

(continued in next column)

28. (BY TI-83) (continued)



which rounds to $x = 0.463$

29. The formula $A = 2000e^{rt}$ can be used to find the dollar value of an investment of \$2000 after t years when the interest is compounded continuously at a rate of r percent.

a) Find the value of the investment after 5 years if the interest rate is 7%

b) Find the value of the investment after 6 years if the interest rate is 5%

Solution:

a) they give us the following:

$$r = 7\% = 0.07$$

$$t = 5$$

so plug these in:

$$A = 2000e^{0.07(5)}$$

“2nd” button

“mode” button

“clear” button

2000

“2nd” button

“ln” button

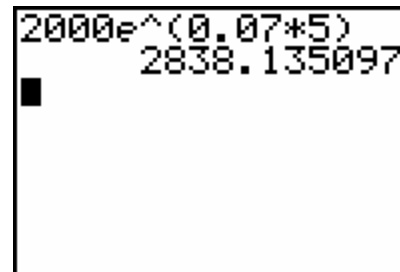
0.07

“multiply” button

5

“)” button

“enter” button



So our answer is \$2838.14

(continued on next page)

29. (continued)

b) they give us the following:

$$r = 5\% = 0.05$$

$$t = 6$$

so plug these in:

$$A = 2000e^{0.05(6)}$$

“2nd” button

“mode” button

“clear” button

2000

“2nd” button

“ln” button

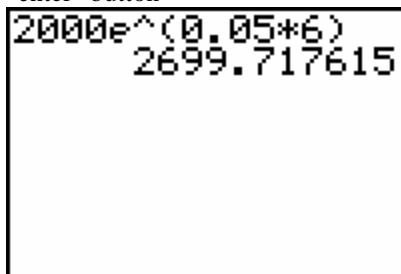
0.05

“multiply” button

6

“)” button

“enter” button



2000e^(0.05*6)
2699.717615

which gives us our answer of:

\$2699.72

30. If a principle of \$630 is invested at an annual interest rate of 5% compounded annually, which is the account balance at the end of 7 years?

Solution:

Since we don't see the word continuous anywhere in the problem, we use the following formula:

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

they give us the following:

$$P = 630$$

$$r = 5\% = 0.05$$

$$n = 1 \text{ (compounded annually)}$$

$$t = 7$$

now plug them in:

$$A = 630 \left(1 + \frac{0.05}{1} \right)^{1(7)}$$

$$A = 886.47$$

31. If \$1550 is invested in an account which earns 7% interest compounded annually, what will be the balance of the account at the end of 11 years? Use the formula: $A = P(1 + r)^t$, where A is the account balance, P is the amount originally invested, r is the interest rate as a decimal, and t is the time invested in years.

Solution:

They give us the following:

$$P = 1550$$

$$r = 7\% = 0.07$$

$$t = 11$$

Now plug them in:

$$A = 1550(1 + 0.07)^{11}$$

$$A = 3262.52$$

32. Find the vertex, focus, and directrix of the parabola:

$$x = \frac{1}{20}(y + 3)^2 - 7$$

Solution:

We need to get it in the form:

$$(y - k)^2 = 4p(x - h)$$

First thing is to flip it around:

$$\frac{1}{20}(y + 3)^2 - 7 = x$$

then:

$$\frac{1}{20}(y + 3)^2 - 7 = x$$

$$\frac{1}{20}(y + 3)^2 = x + 7$$

$$20 \left[\frac{1}{20}(y + 3)^2 \right] = 20(x + 7)$$

$$(y + 3)^2 = 20(x + 7)$$

$$(y - (-3))^2 = 4(5)(x - (-7))$$

so

$$k = -3$$

$$p = 5$$

$$h = -7$$

Now we need to plug them into our formulas:

$$\text{vertex} = (h, k) = (-7, -3)$$

$$\text{focus} = (h + p, k) = (-7 + 5, -3) = (-2, -3)$$

$$\text{directrix} : x = h - p : x = -7 - 5 : x = -12$$

33. Solve by substitution:

$$-8x + 10y = 3$$

$$-x - 7y = 3$$

Solution (BY HAND):

Solve the second equation for x:

$$-x - 7y = 3$$

$$-7y - 3 = x$$

$$x = -7y - 3$$

Now plug this into the other equation:

$$-8x + 10y = 3$$

$$-8(-7y - 3) + 10y = 3$$

$$56y + 24 + 10y = 3$$

$$66y + 24 = 3$$

$$66y = 3 - 24$$

$$66y = -21$$

$$\frac{66y}{66} = \frac{-21}{66}$$

$$y = \frac{-21}{66}$$

$$y = \frac{-7}{22}$$

Now plug this into either equation to find your x
(we will choose the one where we solved for x):

$$x = -7y - 3$$

$$x = -7\left(\frac{-7}{22}\right) - 3$$

$$x = \frac{49}{22} - 3$$

$$x = \frac{49}{22} - \frac{3}{1}$$

$$x = \frac{49}{22} - \frac{66}{22}$$

$$x = \frac{-17}{22}$$

so our answer is:

$$\left(\frac{-17}{22}, \frac{-7}{22}\right)$$

33. Solve by substitution:

$$-8x + 10y = 3$$

$$-x - 7y = 3$$

Solution (BY TI-83):

Lets write down the matrix that this represents:

$$\begin{bmatrix} -8 & 10 & 3 \\ -1 & -7 & 3 \end{bmatrix}$$

(basically we drop all the variables and the equals and leave the numbers and their signs)

Lets enter in the matrix now:

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“enter” on A

2

“enter” (this is the number of rows)

3

“enter” (this is the number of columns)

Now start entering the numbers:

“negative” 8 “enter”

10 “enter”

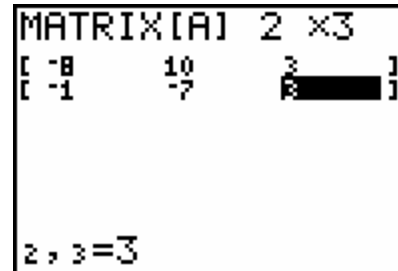
3 “enter”

“negative” 1 “enter”

“negative” 7 “enter”

3 “enter”

This should look like:



Now:

“2nd” button

“mode” button

then

“2nd” button

“x to the negative 1” button

“right arrow” button to MATH

“down arrow” button to RREF

“enter” button

“2nd” button

“x to the negative 1” button

“enter” on A

“)” button

“enter” button

(continued on next page)

33. (BY TI-83) (continued)

```
rref([A])
[[1 0 -.7727272...
 [0 1 -.3181818...
 █
```

since we came up with decimals:

“math” button

“enter” button

“enter” button

```
rref([A])
[[1 0 -.7727272...
 [0 1 -.3181818...
 Ans>Frac
 [[1 0 -17/22]
 [0 1 -7/22 ]]
```

so our answer is:

$$x = \frac{-17}{22}$$

$$y = \frac{-7}{22}$$

or

$$\left(\frac{-17}{22}, \frac{-7}{22} \right)$$

34. Use elimination to solve each system of equations:

$$2x - 3y = -5$$

$$x + 4y = 4$$

Solution (BY HAND):

we can multiply the first equation by 4 and the second equation by 3 to get rid of the y's:

$$4(2x) + 4(-3y) = 4(-5)$$

$$3(x) + 3(4y) = 3(4)$$

gives :

$$8x - 12y = -20$$

$$3x + 12y = 12$$

add *them* :

$$11x = -8$$

$$x = \frac{-8}{11}$$

(continued in next column)

34. (BY HAND) (continued)

$$2x - 3y = -5$$

$$x + 4y = 4$$

we can multiply the second equation by -2 and then add the equations to get rid of the x's:

$$-2(x) - 2(4y) = -2(4)$$

$$-2x - 8y = -8$$

now :

$$2x - 3y = -5$$

$$-2x - 8y = -8$$

add

$$-11y = -13$$

$$\frac{-11y}{-11} = \frac{-13}{-11}$$

$$y = \frac{13}{11}$$

so our answer is:

$$\left(\frac{-8}{11}, \frac{13}{11} \right)$$

34. Use elimination to solve each system of equations:

$$2x - 3y = -5$$

$$x + 4y = 4$$

Solution (BY TI-83):

We first write down the matrix that represents our system of equations:

$$\begin{bmatrix} 2 & -3 & -5 \\ 1 & 4 & 4 \end{bmatrix}$$

then

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“enter” on A

2 “enter” (this is the number of rows)

3 “enter” (this is the number of columns)

Now start entering the numbers:

2 “enter”

“negative” 3 “enter”

“negative” 5 “enter”

1 “enter”

4 “enter”

4 “enter”

This should look like:

(continued on next page)

34. (BY TI-83) (continued)

```
MATRIX[A] 2 x3
[[ 2   -3   -5 ]
 [ 1    4    4 ]]

2, 3=4
```

Now “2nd” “mode” to exit out

Then

“2nd” button

“x to the negative one” button

“right arrow” to EDIT

“down arrow” to RREF

“enter” button

“2nd” button

“x to the negative one” button

“enter” on A

“)” button

“enter” button

```
rref([A])
[[1 0 -.7272727...
 [0 1 1.1818181...
 ■
```

we got decimals so lets change them to fractions:

“math” button

“enter” button

“enter” button

```
rref([A])
[[1 0 -.7272727...
 [0 1 1.1818181...
 Ans>Frac
 [[1 0 -8/11]
 [0 1 13/11]]
```

so our answer is:

$$x = \frac{-8}{11}$$

$$y = \frac{13}{11}$$

so

$$\left(\frac{-8}{11}, \frac{13}{11} \right)$$

35. Solve:

$$x + y - z = 14$$

$$5x - 3y - z = 50$$

$$4x + 2y + 4z = 2$$

Solution (BY HAND):

Step 1: Group the first two equations together:

$$x + y - z = 14$$

$$5x - 3y - z = 50$$

and work on eliminating the z’s. We will multiply the second equation by -1 and then add the two equations together:

$$5x - 3y - z = 50$$

$$-1(5x) - 1(-3y) - 1(-z) = -1(50)$$

$$-5x + 3y + z = -50$$

then

$$x + y - z = 14$$

$$-5x + 3y + z = -50$$

add

$$-4x + 4y = -36$$

can divide everything by 4:

$$-x + y = -9$$

Step 2: Group the second two equations together:

$$5x - 3y - z = 50$$

$$4x + 2y + 4z = 2$$

and work on eliminating the z’s. We will multiply the first equation by 4 and then add the two equations together.

$$4(5x) + 4(-3y) + 4(-z) = 4(50)$$

$$20x - 12y - 4z = 200$$

then

$$20x - 12y - 4z = 200$$

$$4x + 2y + 4z = 2$$

add

$$24x - 10y = 202$$

can divide everything by 2:

$$12x - 5y = 101$$

Step 3: group the equations I found in step 1 and step 2 together:

$$-x + y = -9$$

$$12x - 5y = 101$$

(continued on next page)

35. (BY HAND) (continued)

work on eliminating the y's. Multiply the first equation by 5 and then add them:

$$5(-x) + 5(y) = 5(-9)$$

$$-5x + 5y = -45$$

then

$$-5x + 5y = -45$$

$$12x - 5y = 101$$

$$7x = 56$$

$$\frac{7x}{7} = \frac{56}{7}$$

$$x = 8$$

Step 4: Plug this into one of the equations that had only x's and y's (doesn't matter which one):

$$-x + y = -9$$

$$-8 + y = -9$$

$$y = -9 + 8$$

$$y = -1$$

Step 5: Now plug in the x and y values into one of the original equations:

$$x + y - z = 14$$

$$8 - 1 - z = 14$$

$$7 - z = 14$$

$$7 - 14 = z$$

$$-7 = z$$

so our answer is:

$$(8, -1, -7)$$

35. Solve:

$$x + y - z = 14$$

$$5x - 3y - z = 50$$

$$4x + 2y + 4z = 2$$

Solution (BY TI-83):

First write down the matrix that this gives us:

$$\begin{bmatrix} 1 & 1 & -1 & 14 \\ 5 & -3 & -1 & 50 \\ 4 & 2 & 4 & 2 \end{bmatrix}$$

lets enter this into the calculator:

(continued on next column)

35. (BY TI-83) (continued)

"2nd" button

"x to the negative one" button

"right arrow" twice to EDIT

"enter" on A

3 "enter" (this is the number of rows)

4 "enter" (this is the number of columns)

Now enter the number pressing "enter" after each one. When you get done, it should look like:

"2nd", "mode" to exit out

then

"2nd" button

"x to the negative one" button

"right arrow" to MATH

"down arrow" to RREF

"enter" button

"2nd" button

"x to the negative one" button

"enter" on A

") button

so our answer is:

$$x = 8$$

$$y = -1$$

$$z = -7$$

or

$$(8, -1, -7)$$

36. Solve the system of equations using substitution.

$$x^2 - 4y^2 = -96$$

$$x - 4y = -18$$

Solution (BY HAND):

Take the second equation and solve it for x:

$$x = 4y - 18$$

Now plug this into our first equation:

$$x^2 - 4y^2 = -96$$

$$(4y - 18)^2 - 4y^2 = -96$$

$$(4y - 18)(4y - 18) - 4y^2 = -96$$

$$16y^2 - 72y - 72y + 324 - 4y^2 = -96$$

$$12y^2 - 144y + 324 = -96$$

$$12y^2 - 144y + 324 + 96 = 0$$

$$12y^2 - 144y + 420 = 0$$

divide everything by 12:

$$y^2 - 12y + 35 = 0$$

$$(y - 5)(y - 7) = 0$$

$$y - 5 = 0 \quad y - 7 = 0$$

$$y = 5 \quad y = 7$$

now plug each one in to find x:

$$x = 4y - 18$$

$$\text{for } y = 5$$

$$x = 4(5) - 18 = 20 - 18 = 2$$

$$\text{for } y = 7$$

$$x = 4(7) - 18 = 28 - 18 = 10$$

so

$$(2,5), (10,7)$$

36. Solve the system of equations using substitution.

$$x^2 - 4y^2 = -96$$

$$x - 4y = -18$$

Solution (BY TI-83):

We need to solve each one for y:

(continued in next column)

36. (BY TI-83) (continued)

$$x^2 - 4y^2 = -96$$

$$x^2 + 96 = 4y^2$$

$$\frac{x^2}{4} + \frac{96}{4} = \frac{4y^2}{4}$$

$$\frac{1}{4}x^2 + 24 = y^2$$

$$y^2 = \frac{1}{4}x^2 + 24$$

$$y = \pm \sqrt{\frac{1}{4}x^2 + 24}$$

when solving the first equation for y, we find that we have two equations (one will go into Y1 and one will go into Y2). Keep in mind that even though we put them as two separate equations, they are the same one. Now for the other one:

$$x - 4y = -18$$

$$x + 18 = 4y$$

$$\frac{x + 18}{4} = \frac{4y}{4}$$

$$\frac{x + 18}{4} = y$$

We will put this one into Y3. So:

“y=” button

“clear” button

“2nd” button

“x squared” button

1

“divide” button

4

“x-key” button

“x squared” button

“plus” button

24

“)” button

“down arrow” button

“clear” button

“negative” button

“2nd” button

“x squared” button

1

“divide” button

4

“x-key” button

“x squared” button

“plus” button

24

“)” button

“down arrow” button

“clear” button

(continued on next page)

36. (BY TI-83) (continued)

“(“ button
 “x-key” button
 “plus” button
 18
 “)” button
 “divide” button
 4

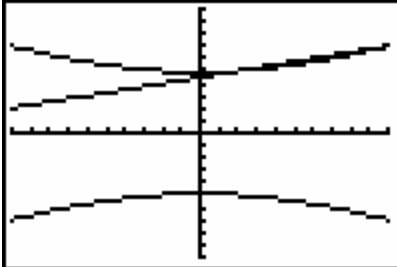
This should look like:

```

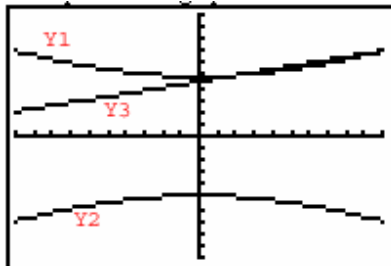
P1ot1 P1ot2 P1ot3
\Y1=√(1/4X²+24)
\Y2=-√(1/4X²+24)

\Y3=(X+18)/4
\Y4=
\Y5=
\Y6=
    
```

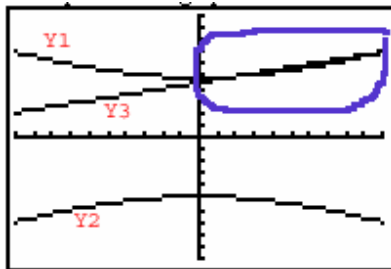
If we press the “graph” button:



If you had watched it while it was graphing, you would see which one was Y1, Y2, and Y3:



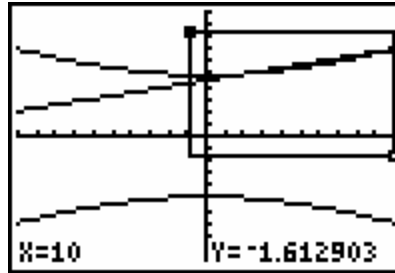
Keep in mind that Y1 and Y2 were the same graph so we are looking for where either one intersects Y3:



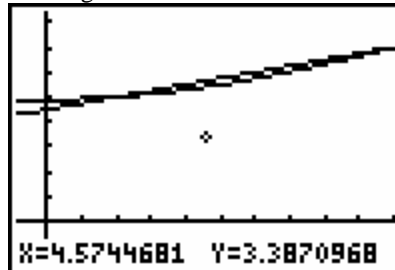
It looks like any intersection is happening in the area circled in blue so lets Zoom-ZoomBox around this area:

(continued in next column)

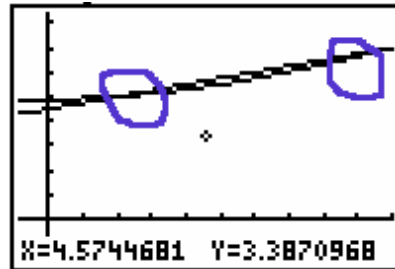
36. (BY TI-83) (continued)



which gives us:

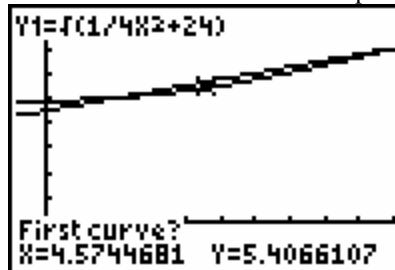


which looks like it intersects at two places:



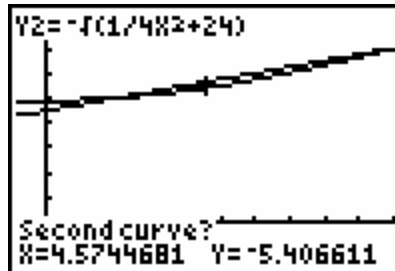
so lets do the intersect method:

“2nd” button
 “trace” button
 “down arrow” to 5:intersect and press “enter”



Notice in the upper left hand corner, it refers to Y1 which is ok since that is one of the ones we want to work with:

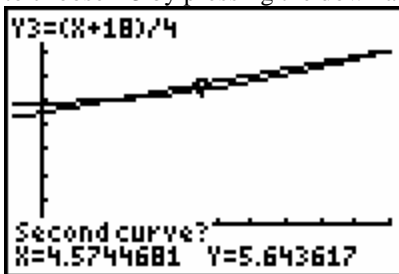
“enter” on First Curve



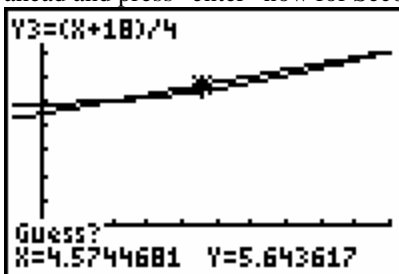
(continued on next page)

36. (BY TI-83) (continued)

Now we see Y2 in the upper left hand corner, we want to see where Y1 intersects with Y3 so we need to choose Y3 by pressing the down arrow:



we can go back and forth between Y1, Y2, and Y3 by pressing the up and down arrow buttons. Go ahead and press "enter" now for Second curve?

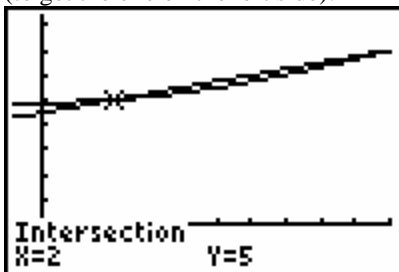


Now:

3

"enter" button

(to get the one on the left side):



so our first answer is (2, 5)

Now we need to get the second answer:

"2nd" button

"trace" button

"down arrow" to 5:intersect and press "enter"

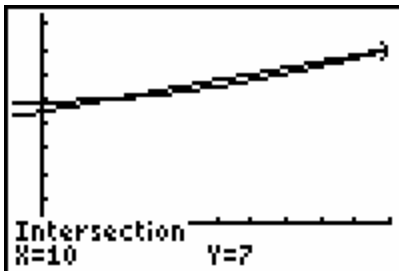
"enter" on First Curve

"down arrow" to choose Y3

"enter" on Second Curve

8

"enter" button



which is our second answer: (10, 7)

37. Find all real solutions to the system of equations using the addition method.

$$x^2 + y^2 = 144$$

$$x^2 - 4y^2 = 64$$

if we multiply the first equation by 4 and then add it to the second one, we will eliminate the y's:

$$4(x^2) + 4(y^2) = 4(144)$$

$$4x^2 + 4y^2 = 576$$

then :

$$4x^2 + 4y^2 = 576$$

$$x^2 - 4y^2 = 64$$

add

$$5x^2 = 640$$

$$\frac{5x^2}{5} = \frac{640}{5}$$

$$x^2 = 128$$

$$x = \pm\sqrt{128}$$

$$x = \pm\sqrt{(2)(8)(8)}$$

$$x = \pm 8\sqrt{2}$$

Now if we take each one of the x-values and plug it in, we can get the y-values:

$$x^2 + y^2 = 144$$

$$(8\sqrt{2})^2 + y^2 = 144$$

$$8^2(\sqrt{2})^2 + y^2 = 144$$

$$64(2) + y^2 = 144$$

$$128 + y^2 = 144$$

$$y^2 = 144 - 128$$

$$y^2 = 16$$

$$y = \pm\sqrt{16}$$

$$y = \pm 4$$

If we put in the negative version, we get the same answers so:

$$(8\sqrt{2}, 4), (8\sqrt{2}, -4), (-8\sqrt{2}, 4), (-8\sqrt{2}, -4)$$

38. Graph the solution set.

$$y \leq -2x + 2$$

$$y < 3$$

Solution: (BY TI-83).

Lets graph each one as if it looked like:

$$y = -2x + 2$$

$$y = 3$$

so

“y=” button

“clear” button

“negative” button

2

“x-key” button

“plus” button

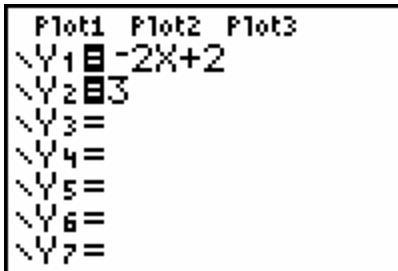
2

“down arrow” button

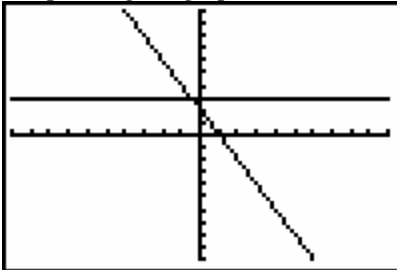
“clear” button

3

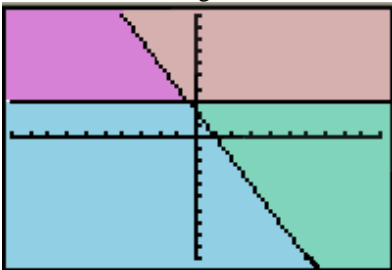
This should look like:



and pressing the graph button:



so we have four regions to look at:



we then chose test cases out of each section:

(5,5) for top right section

(-7, 4) for top left section

(-5, -2) for bottom left section

(6, 0) for bottom right section

We see which sections will work with our equations

(continued in next column)

38. (BY TI-83) (continued)

Since $y < 3$, then:

(5,5) for top right section

(-7, 4) for top left section

the top sections are eliminated because the y-parts are both greater than 3.

Now we are left with the bottom two sections:

(-5, -2) for bottom left section

(6, 0) for bottom right section

these both hold true for $y < 3$ so we have to put them into the first inequality:

$$y \leq -2x + 2$$

$$\text{try: } (-5, -2)$$

$$-2 \leq -2(-5) + 2$$

$$-2 \leq 10 + 2$$

$$-2 \leq 12$$

true

$$\text{try: } (6, 0)$$

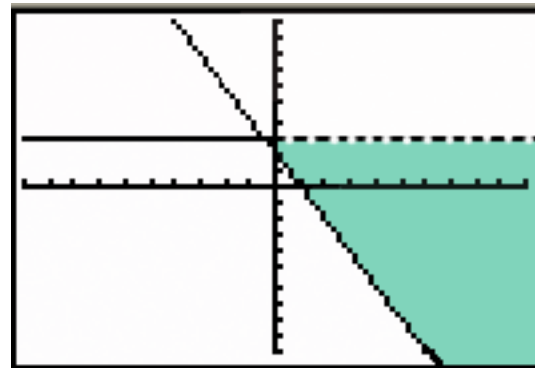
$$0 \leq -2(6) + 2$$

$$0 \leq -12 + 2$$

$$0 \leq -10$$

false

since it was true for (-5, -2) and that is in the bottom right section, that is our answer. There is a less than or equal to on the first equation so that should be a solid line while the other one just has a less than on it so it should be a dashed line.



39. Solve the system of linear equations using Gaussian elimination.

$$13x - 5y = 4$$

$$-2x + 7y = 0$$

Solution (BY HAND):

Write down the matrix that is created from this system of equations:

$$\begin{bmatrix} 13 & -5 & 4 \\ -2 & 7 & 0 \end{bmatrix}$$

Now divide everything in the first row by 13:

$$\begin{bmatrix} \frac{13}{13} & \frac{-5}{13} & \frac{4}{13} \\ -2 & 7 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{-5}{13} & \frac{4}{13} \\ -2 & 7 & 0 \end{bmatrix}$$

Multiply the first row by 2 and add it to the second row: (2R1 + R2 -> R2)

$$2R1 + R2$$

$$= 2 \begin{pmatrix} 1 & \frac{-5}{13} & \frac{4}{13} \end{pmatrix} + (-2 \quad 7 \quad 0)$$

$$= \begin{pmatrix} 2 & \frac{-10}{13} & \frac{8}{13} \end{pmatrix} + (-2 \quad 7 \quad 0)$$

$$= \begin{pmatrix} 2-2 & \frac{-10}{13} + 7 & \frac{8}{13} + 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{81}{13} & \frac{8}{13} \end{pmatrix}$$

so we now have:

$$\begin{bmatrix} 1 & \frac{-5}{13} & \frac{4}{13} \\ 0 & \frac{81}{13} & \frac{8}{13} \end{bmatrix}$$

we want to get a 1 in the second row, second column, so we will multiply everything in row 2 by 13/81 (13/81R2 -> R2)

$$\begin{bmatrix} 1 & \frac{-5}{13} & \frac{4}{13} \\ 0 & 1 & \frac{8}{81} \end{bmatrix}$$

now we want to eliminate the first row, second column so we multiply row 2 by 5/13 and add it to row 1 (5/13R2 + R1 -> R1)

(continued in next column)

39. (BY HAND) (continued)

$$\begin{bmatrix} 1 & 0 & \frac{28}{81} \\ 0 & 1 & \frac{8}{81} \end{bmatrix}$$

answer _ is :

$$\left(\frac{28}{81}, \frac{8}{81} \right)$$

39. Solve the system of linear equations using Gaussian elimination.

$$13x - 5y = 4$$

$$-2x + 7y = 0$$

Solution (BY TI-83):

Write down the matrix that is created from this system of equations:

$$\begin{bmatrix} 13 & -5 & 4 \\ -2 & 7 & 0 \end{bmatrix}$$

we need to enter this matrix in:

“2nd” button

“x to negative one” button

“right arrow” twice to EDIT

“enter” on A

2 “enter” (number of rows)

3 “enter” (number of columns)

Now enter the numbers in:

13 “enter”

“negative” 5 “enter”

4 “enter”

“negative” 2 “enter”

7 “enter”

0 “enter”

“2nd”, “mode” to exit out

Now:

“2nd” button

“x to the negative one” button

“right arrow” to MATH

“down arrow” to RREF

“enter” button

“2nd” button

“x to the negative one” button

“enter” on A

“()” button

“enter” button

(continued on next page)

39. (BY TI-83) (continued)

```
rref([A]
[[1 0 .34567901...
[0 1 .09876543...
█
```

since we have decimals:

“math” button
 “enter” button
 “enter” button

```
rref([A]
[[1 0 .34567901...
[0 1 .09876543...
Ans>Frac
[[1 0 28/81]
[0 1 8/81 ]]
```

so our answer is:

$$\left(\frac{28}{81}, \frac{8}{81}\right)$$

40. Evaluate the expression. $A + B$

$$A = \begin{bmatrix} -8 & -4 & 0 \\ -1 & -3 & 1 \\ -7 & -2 & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -5 & 0 \\ -7 & -9 & 1 \\ -4 & -3 & -8 \end{bmatrix}$$

Solution (BY HAND):

$$A + B$$

$$= \begin{bmatrix} -8 & -4 & 0 \\ -1 & -3 & 1 \\ -7 & -2 & -5 \end{bmatrix} + \begin{bmatrix} -1 & -5 & 0 \\ -7 & -9 & 1 \\ -4 & -3 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -8-1 & -4-5 & 0+0 \\ -1-7 & -3-9 & 1+1 \\ -7-4 & -2-3 & -5-8 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 & 0 \\ -8 & -12 & 2 \\ -11 & -5 & -13 \end{bmatrix}$$

40. Evaluate the expression. $A + B$

$$A = \begin{bmatrix} -8 & -4 & 0 \\ -1 & -3 & 1 \\ -7 & -2 & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -5 & 0 \\ -7 & -9 & 1 \\ -4 & -3 & -8 \end{bmatrix}$$

Solution (BY TI-83):

We need to enter both matrices in:

“2nd” button
 “x to the negative one” button
 “right arrow” twice to EDIT
 “enter” on A
 3 “enter”
 3 “enter”

Now enter all the numbers in A

“2nd”, “mode” to exit out

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“down arrow” to B and press “enter”

3 “enter”

3 “enter”

Now enter all the numbers in B

“2nd”, “mode” to exit out

now

“2nd” button

“x to the negative one”

“enter” on A

“plus” button

“2nd” button

“x to the negative one”

“down arrow to B” and press “enter”

“enter”

```
[A]+[B]
[[ -9  -9  0  ]
 [ -8  -12  2  ]
 [ -11 -5  -13]]
```

(continued on next page)

41. Find the product, if possible: BA

$$A = \begin{bmatrix} -2 & -3 & -2 \\ 1 & -5 & 3 \\ 4 & -4 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} -5 & 5 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & -5 \end{bmatrix}$$

Solution (BY HAND):

BA

$$= \begin{bmatrix} -5 & 5 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} -2 & -3 & -2 \\ 1 & -5 & 3 \\ 4 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5(-2) + 5(1) - 2(4) & -5(-3) + 5(-5) - 2(-4) & -5(-2) + 5(3) - 2(-1) \\ -3(-2) - 4(1) + 1(4) & -3(-3) - 4(-5) + 1(-4) & -3(-2) - 4(3) + 1(-1) \\ -1(-2) - 2(1) - 5(4) & -1(-3) - 2(-5) - 5(-4) & -1(-2) - 2(3) - 5(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 5 - 8 & 15 - 25 + 8 & 10 + 15 + 2 \\ 6 - 4 + 4 & 9 + 20 - 4 & 6 - 12 - 1 \\ 2 - 2 - 20 & 3 + 10 + 20 & 2 - 6 + 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -2 & 27 \\ 6 & 25 & -7 \\ -20 & 33 & 1 \end{bmatrix}$$

41. Find the product, if possible: BA

$$A = \begin{bmatrix} -2 & -3 & -2 \\ 1 & -5 & 3 \\ 4 & -4 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} -5 & 5 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & -5 \end{bmatrix}$$

Solution (BY TI-83):

We need to enter both matrices in:

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“enter” on A

3 “enter”

3 “enter”

Now enter all the numbers in A

“2nd” “mode” to exit out

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“down arrow” to B and press “enter”

3 “enter”

3 “enter”

Now enter all the numbers in B

“2nd” “mode” to exit out

now

“2nd” button

“x to the negative one”

“down arrow” to B

“enter” on B

“2nd” button

“x to the negative one”

“enter” on A

“enter”

42. Find the inverse of the matrix A, if it exists.

$$A = \begin{bmatrix} -2 & 1 & -5 \\ 0 & 4 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

Solution (BY HAND):

Put the identity matrix right next to it:

$$\begin{bmatrix} -2 & 1 & -5 & 1 & 0 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Divide everything in row 1 by -2

Divide everything in row 2 by 4

Divide everything in row 3 by 2

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{5}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{3}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Now do the following operations:

R2 = -3/4R3 + R2

R1 = -5/2R3 + R1

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{5}{4} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{8} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Now do the following operation:

R1 = 1/2R2 + R1

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{8} & -\frac{23}{16} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & -\frac{3}{8} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

so

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{8} & -\frac{23}{16} \\ 0 & \frac{1}{4} & -\frac{3}{8} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

42. Find the inverse of the matrix A, if it exists.

$$A = \begin{bmatrix} -2 & 1 & -5 \\ 0 & 4 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

Solution (BY TI-83):

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“enter” on A

3 “enter”

3 “enter”

Now enter all the numbers in A

“2nd” “mode” to exit out

then

“2nd” button

“x to the negative one” button

“enter” on A

“x to the negative one” button

“enter” button

[A]⁻¹
 [[[-.5 .125 -1.4...
 [0 .25 -.37...
 [0 0 .5 ...

we have decimals so:

“math” button

“enter” button

“enter” button

Ans>Frac
 [[[-1/2 1/8 -23/...
 [0 1/4 -3/8...
 [0 0 1/2 ...

we need to “right arrow” to see what is to the left.

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{8} & -\frac{23}{8} \\ 0 & \frac{1}{4} & -\frac{3}{8} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

43. If possible, solve the system of equations using an inverse matrix.

$$-2x + 4y = 0$$

$$5x - 8y = -7$$

Solution: (BY HAND)

First identify our matrices:

$$A = \begin{bmatrix} -2 & 4 \\ 5 & -8 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

$$X = A^{-1}b$$

so our first step is find the inverse of A:

$$\begin{bmatrix} -2 & 4 & 1 & 0 \\ 5 & -8 & 0 & 1 \end{bmatrix}$$

$$R1 = -1/2R1$$

$$\begin{bmatrix} 1 & -2 & \frac{-1}{2} & 0 \\ 5 & -8 & 0 & 1 \end{bmatrix}$$

$$R2 = -5R1 + R2$$

$$\begin{bmatrix} 1 & -2 & \frac{-1}{2} & 0 \\ 0 & 2 & \frac{5}{2} & 1 \end{bmatrix}$$

$$R2 = 1/2R2$$

$$\begin{bmatrix} 1 & -2 & \frac{-1}{2} & 0 \\ 0 & 1 & \frac{5}{4} & \frac{1}{2} \end{bmatrix}$$

$$R1 = 2R2 + R1$$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & \frac{5}{4} & \frac{1}{2} \end{bmatrix}$$

so:

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ \frac{5}{4} & \frac{1}{2} \end{bmatrix}$$

(continued on next page)

43. (BY HAND) (Continued)

now

X

$$= A^{-1}b$$

$$= \begin{bmatrix} 2 & 1 \\ 5 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

$$= \begin{bmatrix} 2(0) + 1(-7) \\ \frac{5}{4}(0) + \frac{1}{2}(-7) \end{bmatrix}$$

$$= \begin{bmatrix} -7 \\ -\frac{7}{2} \end{bmatrix}$$

43. If possible, solve the system of equations using an inverse matrix.

$$-2x + 4y = 0$$

$$5x - 8y = -7$$

Solution: (BY TI-83)

First lets write down the matrix that this creates:

$$\begin{bmatrix} -2 & 4 & 0 \\ 5 & -8 & -7 \end{bmatrix}$$

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“enter” on A

2 “enter”

3 “enter”

Now enter all the numbers in A

“2nd”, “mode” to exit out

“2nd” button

“x to the negative one” button

“right arrow” to MATH

“down arrow” to RREF

“enter” button

“2nd” button

“x to the negative one” button

“enter” on A

“)” button

“enter” button

we have decimals so:

“math” button

“enter” button

“enter” button

which gives our answer:

$$\left(-7, \frac{-7}{2} \right)$$

44. Find the determinant of the matrix:

$$\begin{bmatrix} -3 & -5 \\ -4 & -1 \end{bmatrix}$$

Solution (BY HAND):

$$(-3)(-1) - (-5)(-4)$$

$$3 - 20$$

$$-17$$

44. Find the determinant of the matrix:

$$\begin{bmatrix} -3 & -5 \\ -4 & -1 \end{bmatrix}$$

Solution (BY TI-83):

“2nd” button

“x to the negative one” button

“right arrow” twice to EDIT

“enter” on A

2 “enter”

2 “enter”

Now enter all the numbers in A

“2nd”, “mode” to exit out

“2nd” button

“x to the negative one” button

“right arrow” to MATH

“enter” on DET(

“2nd” button

“x to the negative one” button

“enter” on A

“)” button

“enter” button

gives you the answer

$$-17$$

45. Use Cramer’s Rule to solve (if possible) the system of equations

$$2x + 5y = 10$$

$$3x - 3y = 7$$

Solution (BY HAND):

We need to find the determinants:

$$D = \begin{vmatrix} 2 & 5 \\ 3 & -3 \end{vmatrix} = -21$$

$$D_x = \begin{vmatrix} 10 & 5 \\ 7 & -3 \end{vmatrix} = -65$$

$$D_y = \begin{vmatrix} 2 & 10 \\ 3 & 7 \end{vmatrix} = -16$$

(continued on next page)

45. (BY HAND) (continued)

$$x = \frac{D_x}{D} = \frac{-65}{-21} = \frac{65}{21}$$

$$y = \frac{D_y}{D} = \frac{-16}{-21} = \frac{16}{21}$$

$$\left(\frac{65}{21}, \frac{16}{21} \right)$$

45. Use Cramer's Rule to solve (if possible) the system of equations

$$2x + 5y = 10$$

$$3x - 3y = 7$$

Solution (BY TI-83):

"2nd" button

"x to the negative one" button

"right arrow" twice to EDIT

"enter" on A

2 "enter"

3 "enter"

Now enter all the numbers in A

"2nd" "mode" to exit out

"2nd" button

"x to the negative one" button

"right arrow" to MATH

"down arrow" to RREF

"enter" button

"2nd" button

"x to the negative one" button

"enter" on A

")" button

"enter" button

we have decimals so:

"math" button

"enter" button

"enter" button

which gives our answer:

$$\left(\frac{65}{21}, \frac{16}{21} \right)$$

46. Find the minor M_{32} of the matrix A

$$A = \begin{bmatrix} 4 & -2 & -9 \\ 3 & -8 & 7 \\ 5 & -6 & 1 \end{bmatrix}$$

Solution:

(continued on next page)

46. (continued)

$$\begin{bmatrix} 4 & -2 & -9 \\ 3 & -8 & 7 \\ 5 & -6 & 1 \end{bmatrix}$$

so we are finding the following determinant:

$$\begin{vmatrix} 4 & -9 \\ 3 & 7 \end{vmatrix}$$

$$= (4)(7) - (3)(-9)$$

$$= 28 + 27$$

$$= 55$$

47. Solve by factoring.

$$x^2 + x - 72 = 0$$

solution:

	72	
P	S	D
(1)(72)	73	71
(2)(36)	38	34
(3)(24)	27	21
(4)(18)	22	14
(6)(12)	18	6
(8)(9)	17	1

so

$$(x + 9)(x - 8) = 0$$

$$x + 9 = 0 \quad x - 8 = 0$$

$$x = -9 \quad x = 8$$

48. Solve by taking the square root:

$$3x^2 = 12$$

Solution:

$$3x^2 = 12$$

$$\frac{3x^2}{3} = \frac{12}{3}$$

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

49. Solving using the quadratic formula:

$$5x^2 - 2x - 5 = 0$$

Solution:

$$5x^2 - 2x - 5 = 0$$

$$a = 5, b = -2, c = -5$$

so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-5)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{4 + 100}}{10}$$

$$x = \frac{2 \pm \sqrt{104}}{10}$$

$$x = \frac{2 \pm \sqrt{(2)(2)(26)}}{10}$$

$$x = \frac{2 \pm 2\sqrt{26}}{10}$$

$$x = \frac{1 \pm \sqrt{26}}{5}$$

$$x = \frac{1 \pm \sqrt{26}}{5}$$

50. Find the vertex of the graph of the function

$$f(x) = (x + 2)^2 + 1$$

Solution:

Compare it to our standard form:

$$f(x) = a(x - h)^2 + k$$

$$\text{vertex} = (h, k)$$

then

$$f(x) = 1(x - (-2))^2 + 1$$

so

$$h = -2$$

$$k = 1$$

$$\text{vertex} = (-2, 1)$$

