

## Estimating a Population Mean

## Student's t-Distribution

Suppose that a simple random sample of size  $n$  is taken from a population. If the population from which the sample is drawn follows a normal distribution, the distribution of:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

follows Student's t-distribution with  $n - 1$  degrees of freedom.

## Assumptions

1. sample data come from a simple random sample or randomized experiment
2. sample size is small relative to the population size ( $n \leq 0.05N$ )
3. the data comes from a population that is normally distributed, or the sample size is large

## Finding $t_{\alpha/2}$

1. Find the area in one tail
2. Find Degree of Freedom:  $DF = n - 1$
3. Look up the value in t-distribution table

1. Find the t-value such that the area in the right tail is 0.10 with 12 degrees of freedom

2. Find the t-value such that the area in the right tail is 0.05 with 20 degrees of freedom

3. Find the t-value such that the area left of the t-value is 0.01 with 9 degrees of freedom

4. Find the t-value that corresponds to 90% confidence. Assume 15 degrees of freedom

### Confidence Interval

$$\text{Lower bound: } \bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$\text{Upper bound: } \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$\bar{x}$  is point estimate

### TI-83/84 Instructions

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Finding confidence intervals

1. "Stat" button
2. Choose "Tests" Menu
3. Choose "TInterval"
4. Highlight "Stats" (or "Data" if given raw data)
5. Enter std dev, mean, n, and C-level
6. "Highlight Calculate" and press "Enter"

$$E = UB - \bar{x}$$

5. Determine the point estimate of the population mean and margin of error for each confidence interval

Lower bound: 20, upper bound: 28

### 6. Confidence Interval (By Hand and By TI-83/84)

A simple random sample of size 25 has a sample mean of 20.2 and a sample standard deviation of 2.1, construct a 95% confidence interval for the population mean (assume data is normally distributed)

### 7. Confidence Interval (By Hand and By TI-83/84)

Ages of students at the college follow a normal distribution. If a sample of 15 students has an average age of 18.2 with a standard deviation of 0.5. Construct a 99% confidence interval for the population mean

### 8. Confidence Interval (By Hand and By TI-83/84)

A sample of scores are listed below (assume the scores are normally distributed), construct a 90% interval for the population mean:

80 82 82 84 90 95 97 97

### Sample Size to Estimate Population Mean

$$n = \left[ \frac{z_{\alpha/2} \cdot S}{E} \right]^2$$

$z_{\alpha/2}$  = critical z score based on desired degree of confidence

E = desired margin of error

S = previous sample standard deviation

### 9. Sample size

If we wish to estimate the mean age of students at the college with 95% confidence within 0.2 years, how many students should we sample assuming the sample standard deviation from last year was 1.3?

### 10. Sample size

If we wish to estimate the average hours that students watch television at 99% confidence within 1 hour, how many students should we sample assuming the sample standard deviation from last year was 2.1?