Estimating a Population Standard Deviation

Chi-Square Distribution

If a simple random sample of size n is obtained from a normally distributed population with mean μ and standard deviation σ, then:

$$X^2 = \frac{(n-1)s^2}{\sigma^2}$$

has a chi-square distribution with n – 1 degrees of freedom

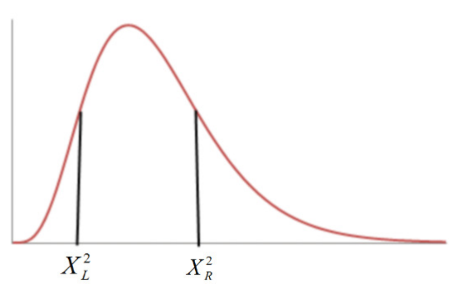
Characteristics of the Chi-Square Distribution

1. It is not symmetric
2. The shape of the chi-square distribution depends on the degrees of freedom, just like the Student's t-distribution
3. As the number of degrees of freedom increases, the chi-square distribution becomes more nearly symmetric
4. The values of χ^2 are nonnegative

Note

We use chi-square distribution when finding confidence intervals of population variance and standard deviation

Critical Values



Finding Critical Values (left chi square and right chi square)

1. Find Degree of freedom (DF) = n – 1

2. If given the percentage, find the amount in each tail:

example, if I have 90% degree of confidence, I would have 5% in each tail.

Finding Critical Values (left chi square and right chi square) - Continued

3. Find the lookup value:

For the right tail, I would look at the area to the right of the 90% in the middle, in this case it would be 5% or 0.05

For the left tail, I would look at the area to the right of the 5% on the left side, in this case it would be 95% or 0.95

Finding Critical Values (left chi square and right chi square) - Continued

4. Looking at the Chi Square Distribution: using the DF and the lookup value, find the appropriate value of the right and left critical values

1. Critical Values

Given 90% confidence desired and n = 15, find the critical values

2. Critical Values

Given 95% confidence desired and n = 21, find the critical values

Confidence Interval for Population Standard Deviation

$$\text{Lower Bound} = \sqrt{\frac{(n-1)s^2}{X_R^2}}$$

$$\text{Upper Bound} = \sqrt{\frac{(n-1)s^2}{X_L^2}}$$

Confidence Interval for Population Variance

$$\text{Lower Bound} = \frac{(n-1)s^2}{X_R^2}$$

$$\text{Upper Bound} = \frac{(n-1)s^2}{X_L^2}$$

3. Confidence Intervals

Given grades are normally distributed at a college, if a sample of 18 grades has a sample standard deviation of 2.5, construct a 95% confidence interval for the population standard deviation for the grades

4. Confidence Intervals

Given ages are normally distributed in a particular town, if a sample of 25 people in the town have a sample standard deviation of 10.1, construct a 99% confidence interval for the population variance for the ages in this town

5. Confidence Intervals

A sample of stock prices in a particular industry revealed the following stock prices:

15 18 17 13 14 16 15 13

Construct a 90% confidence interval for the population standard deviation of stock prices in this industry

Comparison in Notation

$$\chi_L^2 = \chi_{1-\alpha/2}^2$$

$$\chi_R^2 = \chi_{\alpha/2}^2$$