

Hypothesis Tests for a Population Standard Deviation

Chi-Square Distribution

If a simple random sample of size n is obtained from a normally distributed population with mean μ and standard deviation σ , then:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

has a chi-square distribution with $n - 1$ degrees of freedom

Characteristics of the Chi-Square Distribution

1. It is not symmetric
2. The shape of the chi-square distribution depends on the degrees of freedom, just like the Student's t-distribution
3. As the number of degrees of freedom increases, the chi-square distribution becomes more nearly symmetric
4. The values of χ^2 are nonnegative

Requirements

- the sample is obtained using simple random sampling
- the population is normally distributed.

Classical Approach (By Hand)

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis (H_0 always has the equals part on it)
3. See if claim matches H_0 or H_1
4. Draw the picture and split α into tails
 - $H_1: \sigma \neq \text{value}$ Two Tail
 - $H_1: \sigma < \text{value}$ Left Tail
 - $H_1: \sigma > \text{value}$ Right Tail

Classical Approach (By Hand) (cont.)

5. Find critical values: Use Chi-Square Distribution table
6. Find test statistic: $\chi_o^2 = \frac{(n-1)s^2}{\sigma_o^2}$
7. If test statistic falls in tail, Reject H_0 . If test statistic falls in main body, Accept H_0 . Determine the claim based on step 3

1. Age of Students

If ages of students at a campus are normally distributed with a mean of 20 and a standard deviation of 2.1 It is believed that the standard deviation is lower now so a study is done asking 15 students their age. It is found that the standard deviation is 1.8. Test the claim at $\alpha = 0.05$.

2. Age of Students

If ages of students at a campus are normally distributed with a mean of 19 and a standard deviation of 1.3 It is believed that the standard deviation is different now so a study is done asking 22 students their age. It is found that the standard deviation is 1.9. Test the claim at $\alpha = 0.10$.

3. Age of Students

If ages of students at a campus are normally distributed with a mean of 23 and a standard deviation of 3.4 It is believed that the standard deviation is more now so a study is done asking 12 students their age. It is found that the standard deviation is 3.5. Test the claim at $\alpha = 0.01$.

Note

- If s (sample standard deviation) is not given in a problem, you will have to find it with one of the following:
 - 1-VARSTATS on a TI-83/84

$$- s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$