Hypothesis Tests for a Population
Standard Deviation

Chi-Square Distribution
If a simple random sample of size \( n \) is obtained from a normally distributed population with mean \( \mu \) and standard deviation \( \sigma \), then:

\[
\chi^2 = \frac{(n - 1)s^2}{\sigma^2}
\]

has a chi-square distribution with \( n - 1 \) degrees of freedom.

Characteristics of the Chi-Square Distribution
1. It is not symmetric
2. The shape of the chi-square distribution depends on the degrees of freedom, just like the Student's t-distribution
3. As the number of degrees of freedom increases, the chi-square distribution becomes more nearly symmetric
4. The values of \( \chi^2 \) are nonnegative

Requirements
• the sample is obtained using simple random sampling
• the population is normally distributed.

Classical Approach (By Hand)
1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis (\( H_0 \) always has the equals part on it)
3. See if claim matches \( H_0 \) or \( H_1 \)
4. Draw the picture and split \( \alpha \) into tails
   \( H_1: \sigma \neq \text{value} \) Two Tail
   \( H_1: \sigma < \text{value} \) Left Tail
   \( H_1: \sigma > \text{value} \) Right Tail

Classical Approach (By Hand) (cont.)
5. Find critical values: Use Chi-Square Distribution table
6. Find test statistic: \( \chi^2 = \frac{(n-1)s^2}{\sigma^2} \)
7. If test statistic falls in tail, Reject \( H_0 \). If test statistic falls in main body, Accept \( H_0 \). Determine the claim based on step 3
1. Age of Students
If ages of students at a campus are normally distributed with a mean of 20 and a standard deviation of 2.1. It is believed that the standard deviation is lower now so a study is done asking 15 students their age. It is found that the standard deviation is 1.8. Test the claim at $\alpha = 0.05$.

2. Age of Students
If ages of students at a campus are normally distributed with a mean of 19 and a standard deviation of 1.3. It is believed that the standard deviation is different now so a study is done asking 22 students their age. It is found that the standard deviation is 1.9. Test the claim at $\alpha = 0.10$.

3. Age of Students
If ages of students at a campus are normally distributed with a mean of 23 and a standard deviation of 3.4. It is believed that the standard deviation is more now so a study is done asking 12 students their age. It is found that the standard deviation is 3.5. Test the claim at $\alpha = 0.01$.

Note
• If $s$ (sample standard deviation) is not given in a problem, you will have to find it with one of the following:
  − 1-VARSTATS on a TI-83/84
  − $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$