

## Inference about Two Means: Dependent Samples

### Testing Hypothesis Regarding the Difference of Two Means Using a Matched-Pairs Design: Requirements

- Sample is obtained using simple random sampling or through a randomized experiment
- Sample data are matched pairs
- Differences are normally distributed with no outliers or the sample size,  $n$ , is large ( $n \geq 30$ )
- Sampled values are independent (sample size is no more than 5% of population size)

#### Classical Approach (TI-83/84)

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis ( $H_0$  always has the equals part on it)
3. See if claim matches  $H_0$  or  $H_1$
4. Draw the picture and split  $\alpha$  into tail(s)
  - $H_1: \mu_d \neq 0$  Two Tail
  - $H_1: \mu_d < 0$  Left Tail
  - $H_1: \mu_d > 0$  Right Tail
5. Put first list in L1, second list in L2, highlight L3 and enter "L1-L2", exit out
6. Find critical values (t-Distribution table)
7. Find test statistic (T-TEST)
8. If test statistic falls in tail, Reject  $H_0$ . If test statistic falls in main body, Accept  $H_0$ . Determine the claim based on step 3

#### Classical Approach (By Hand)

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis ( $H_0$  always has the equals part on it)
3. See if claim matches  $H_0$  or  $H_1$
4. Draw the picture and split  $\alpha$  into tails
  - $H_1: \mu_d \neq 0$  Two Tail
  - $H_1: \mu_d < 0$  Left Tail
  - $H_1: \mu_d > 0$  Right Tail
5. Find critical values: Use t-Distribution table

#### Classical Approach (By Hand) (cont.)

5. Subtract the values in the second list from the first list and find the mean ( $\bar{d}$ ) and the sample standard deviation ( $s_d$ ) of the differences
6. Find test statistic:  $t_o = \frac{\bar{d}-0}{\frac{s_d}{\sqrt{n}}}$
7. If test statistic falls in tail, Reject  $H_0$ . If test statistic falls in main body, Accept  $H_0$ . Determine the claim based on step 3

#### P-Value Approach (TI-83/84)

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis ( $H_0$  always has the equals part on it)
3. See if claim matches  $H_0$  or  $H_1$
4. Put first list in L1, second list in L2, highlight L3 and enter "L1-L2", exit out
5. Find p-value (T-TEST)
5. If p-value is less than  $\alpha$ , Reject  $H_0$ . If p-value is greater than  $\alpha$ , Accept  $H_0$ . Determine the claim based on step 3

### P-Value Approach (By Hand)

1. Write down a shortened version of claim
2. Come up with null and alternate hypothesis ( $H_0$  always has the equals part on it)
3. See if claim matches  $H_0$  or  $H_1$
4. Subtract the values in the second list from the first list and find the mean ( $\bar{d}$ ) and the sample standard deviation ( $s_d$ ) of the differences

5. Find test statistic:  $t_o = \frac{\bar{d}-0}{\frac{s_d}{\sqrt{n}}}$

### P-Value Approach (By Hand) (cont.)

6. Lookup the absolute value of the t-score from step 4 in the t-Distribution table and find the p-value(s) from table (will be some kind of range). If it is 2 tails, then multiply the p-value(s) by 2.
7. If p-value is less than  $\alpha$ , Reject  $H_0$ . If p-value is greater than  $\alpha$ , Accept  $H_0$ . Determine the claim based on step 3

### 1. Claim

Given reaction time to red and reaction time to blue, is the reaction time to red greater than the reaction time to blue? Use the matched pairs data below at  $\alpha = 0.05$ . (Assume the differences are approximately normal with no outliers)

	1	2	3	4	5	6
Red	0.425	0.511	0.712	0.651	0.499	0.505
Blue	0.404	0.622	0.605	0.719	0.813	0.350

### Constructing a Confidence Interval for the Difference between Two Population Means (Matched Pairs)

Confidence interval for  $\mu_d$  is given by:

$$\text{Lower bound: } \bar{d} - t_{\alpha/2} \left( \frac{s_d}{\sqrt{n}} \right)$$

$$\text{Upper bound: } \bar{d} + t_{\alpha/2} \left( \frac{s_d}{\sqrt{n}} \right)$$

Where  $\bar{d}$  and  $s_d$  are the mean and standard deviation of the differenced data

### TI-83/84 Instructions

1. Put first list in L1, second list in L2, highlight L3 and enter "L1-L2", exit out
2. STAT Button, Right Arrow to TESTS, Choose Tinterval
3. Choose Data, L3, 1, and confidence desired and then Calculate

This will give you the interval.

$$\text{Point Estimate} = \frac{UB - LB}{2}$$

$$\text{Margin of Error (E)} = UB - \text{Point Estimate}$$

### 2. Confidence Intervals

Given reaction time to red and reaction time to blue, Create a 99% confidence interval about the population mean difference between red and blue.

	1	2	3	4	5	6
Red	0.425	0.511	0.712	0.651	0.499	0.505
Blue	0.404	0.622	0.605	0.719	0.813	0.350