

## Measures of Position and Outliers

z-score (standard score) = number of standard deviations that a given value is above or below the mean (Round z to two decimal places)

Sample z - score

$$z = \frac{x - \bar{x}}{s}$$

Population z - score

$$z = \frac{x - \mu}{\sigma}$$

Ordinary values: between and including -2 and 2  
Unusual values: less than -2 and greater than 2

### 1. Determine which student did better

In the 8:00 college algebra class, on the first test the mean was 75 with a standard deviation of 3, Bill was in this class and got a 82. In the 10:00 college algebra class, on the first test the mean was 83 with a standard deviation of 2, Sally was in this class and got 83. Did Sally or Bill do better relative to their classes?

### kth Percentile ( $P_k$ )

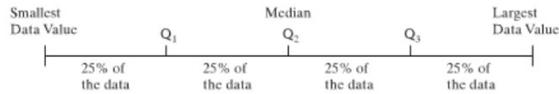
The kth percentile of a set of data is a value such that k percent of the observations are less than or equal to the value

\*Quartiles : Divides the data into four equal parts ( $Q_1, Q_2, Q_3$ )

\*Deciles : Divides the data into ten equal parts ( $D_1, D_2, \dots, D_9$ )

\*Percentiles : Divides the data into 100 equal parts ( $P_1, \dots, P_{99}$ )

## Quartiles



## Finding Quartiles (By Hand)

1. Arrange the data in ascending order
2. Determine the median, M, or second quartile, Q<sub>2</sub>
3. Divide the data set into halves: the observations below (to the left of) M and the observations above M. The first quartile, Q<sub>1</sub>, is the median of the bottom half and the third quartile, Q<sub>3</sub>, is the median of the top half

### 2. Find the quartiles of the following data

33, 37, 43, 51, 62, 65, 72, 83, 91, 105, 110, 201, 305, 317, 320, 330

## Find Quartiles (By TI-83/84)

1. Input numbers, then “2<sup>nd</sup>” “mode” to exit out
2. “stat” button, “right arrow” to CALC, “enter” on 1-varstats, “enter”

Note: down arrow to see quartiles

### 3. Find the quartiles of the following data

2, 8, 33, 45, 71, 83, 91, 105, 113, 235, 471, 512, 513, 555, 1002

## Finding the percentile of a number in a list (sorted)

$$\frac{\text{number of values less than } x}{\text{total number of values}} \cdot 100$$

where x is a number in our list

#### 4. Find the percentile of a number in a list

Given:

15 18 23 24 27  
33 38 41 52 63  
71 81 88 91 99

Find the percentile of 33

#### Find the nth Percentile ( $P_k$ )

1. Sort the data
2. Compute the Locator Value:  $i = (k/100)(n+1)$   
 $k$ =percentile in question  
 $n$ =number of values

Note:  $i$  is the position of your answer in the list

1. If  $i$  has no decimals:  
Percentile =  $i$ th value  
If it has decimals:  
Percentile = Average of the two numbers it falls between (position wise).

#### 5. Find the nth percentile

Given:

17 21 25 31 43  
55 71 83 91 105  
210 233 301 315 320

Find  $P_{25}$

#### 6. Find the nth percentile

Given:

17 21 25 31 43  
55 71 83 91 105  
210 233 301 315 320

Find  $P_{17}$

#### Quartiles to Percentiles

- 25<sup>th</sup> percentile = 1<sup>st</sup> quartile
- 50<sup>th</sup> percentile = 2<sup>nd</sup> quartile
- 75<sup>th</sup> percentile = 3<sup>rd</sup> quartile

#### Other Formulas

- Interquartile range (IQR) =  $Q_3 - Q_1$
- Semi-interquartile range =  $(Q_3 - Q_1) / 2$
- Midquartile =  $(Q_3 + Q_1) / 2$
- 10-90 percentile range =  $P_{90} - P_{10}$

**SUMMARY: WHICH MEASURES TO REPORT**

Shape of Distribution	Measure of Central Tendency	Measure of Dispersion
Symmetric	Mean	Standard deviation
Skewed left or skewed right	Median	Interquartile range

## Definitions

- Outliers = Extreme observations

## Checking for Outliers by Using Quartiles

1. Determine Q1 and Q3
2. Find  $IQR = Q3 - Q1$
3. Determine the fences (fences serve as cutoff points for determining outliers)  
Lower Fence =  $Q1 - 1.5(IQR)$   
Upper Fence =  $Q3 + 1.5(IQR)$
4. Outliers are values less than lower fence or greater than upper fence

7. Given the population data below, find the following:

- a) Compute the z-score corresponding to 5.3
- b) Determine the quartiles
- c) Compute the interquartile range (IQR)
- d) Determine the lower and upper fences, are there any outliers?

3.5	3.2	4.1	4.7	3.8	5.1
7.2	3.9	5.3	6.1	6.2	3.1
9.3	9.4	10.5	10.9	11.1	11.2
12.7	12.8	13.1	13.5	17	38.6