The Probability of a Type II Error and the Power of the Test

Probability of a Type II Error / Power of Test

- Type I Error: Rejecting Null Hypothesis when Null Hypothesis is true (α is the probability of a Type I Error)
- Type II Error: Accepting Null Hypothesis when Null Hypothesis is false.

Review: How to Determine Picture

H_1: p ≠ value Two Tails

H_1: p < value Left Tail

H_1: p > value Right Tail

Finding Probability of a Type II Error / Power of Test (Proportion) (TI-83/84)

1. Write down claim
2. Write down null and alternate hypothesis
3. Draw picture (determine tail(s))
4. Find ZA_L and/or ZA_R (you will have both for two tails) – round to 3 decimal places
   Left Tail: ZA_L = INVNORM(α)
   Right Tail: ZA_R = INVNORM(1-α)
   Two Tails: ZA_L = INVNORM(α/2)
   and ZA_R = INVNORM(1-α/2)

Finding Probability of a Type II Error / Power of Test (Proportion) (TI-83/84)

5. Find $\hat{p}_L$ and/or $\hat{p}_R$ (you will have both for two tails) – round to 3 decimal places

- Left Tail: $\hat{p}_L = p_o + ZA_L\sqrt{\frac{p_o(1-p_o)}{n}}$
- Right Tail: $\hat{p}_R = p_o + ZA_R\sqrt{\frac{p_o(1-p_o)}{n}}$
- Two Tails: $\hat{p}_L = p_o + ZA_L\sqrt{\frac{p_o(1-p_o)}{n}}$

  and $\hat{p}_R = p_o + ZA_R\sqrt{\frac{p_o(1-p_o)}{n}}$

p_o is the number from the claim (in decimal form)

Finding Probability of a Type II Error / Power of Test (Proportion) (TI-83/84)

6. Find z_L and/or z_R (you will have both for two tails) – round to 2 decimal places

- Left Tail: $z_L = \frac{\hat{p}_L - p}{\sqrt{\frac{p(1-p)}{n}}}$
- Right Tail: $z_R = \frac{\hat{p}_R - p}{\sqrt{\frac{p(1-p)}{n}}}$
- Two Tails: $z_L = \frac{\hat{p}_L - p}{\sqrt{\frac{p(1-p)}{n}}}$ and $z_R = \frac{\hat{p}_R - p}{\sqrt{\frac{p(1-p)}{n}}}$

p is the true population proportion (in decimal form)

Finding Probability of a Type II Error / Power of Test (Proportion) (TI-83/84)

7. Find the Probability of a Type II Error (β)
- Left Tail: β = NORMALCDF(z_L,E99)
- Right Tail: β = NORMALCDF(-E99,z_R)
- Two Tails: β = NORMALCDF(z_L,z_R)

8. Find the power of the test (1-β)

Finding Probability of a Type II Error / Power of Test (Proportion) (By Hand)

1. Write down claim
2. Write down null and alternate hypothesis
3. Draw picture (determine tail(s))
4. Find ZA_L and/or ZA_R (you will have both for two tails) – Use Standard Normal Distribution table
   - Left Tail: ZA_L = Look up α
   - Right Tail: ZA_R = Look up (1-α)
   - Two Tails: ZA_L = Look up (α/2) and ZA_R = Look up (1-α/2)

Finding Probability of a Type II Error / Power of Test (Proportion) (By Hand)

5. Find $\hat{p}_L$ and/or $\hat{p}_R$ (you will have both for two tails) – round to 3 decimal places

- Left Tail: $\hat{p}_L = p_o + ZA_L\sqrt{\frac{p_o(1-p_o)}{n}}$
- Right Tail: $\hat{p}_R = p_o + ZA_R\sqrt{\frac{p_o(1-p_o)}{n}}$
- Two Tails: $\hat{p}_L = p_o + ZA_L\sqrt{\frac{p_o(1-p_o)}{n}}$ and $\hat{p}_R = p_o + ZA_R\sqrt{\frac{p_o(1-p_o)}{n}}$

p_o is the number from the claim (in decimal form)

Finding Probability of a Type II Error / Power of Test (Proportion) (By Hand)

6. Find z_L and/or z_R (you will have both for two tails) – round to 2 decimal places

- Left Tail: $z_L = \frac{\hat{p}_L - p}{\sqrt{\frac{p(1-p)}{n}}}$
- Right Tail: $z_R = \frac{\hat{p}_R - p}{\sqrt{\frac{p(1-p)}{n}}}$
- Two Tails: $z_L = \frac{\hat{p}_L - p}{\sqrt{\frac{p(1-p)}{n}}}$ and $z_R = \frac{\hat{p}_R - p}{\sqrt{\frac{p(1-p)}{n}}}$

p is the true population proportion (in decimal form)

Finding Probability of a Type II Error / Power of Test (Proportion) (By Hand)

7. Find the Probability of a Type II Error (β). Look up z scores in standard normal distribution table.
- Left Tail: β = 1 – (value from table based on z_L)
- Right Tail: β = (value from table based on z_R)
- Two Tails: β = (value from table based on z_R) – (value from table based on z_L)

8. Find the power of the test (1-β)

1. Find Probability of Type II Error / Power of Test

To test Ho: p = 0.40 versus H1: p < 0.40, a simple random sample of n = 200 is obtained and 90 successes are observed. If the researcher decides to test this hypothesis at the α = 0.05 level of significance, compute the probability of making a Type II Error if the true population proportion is 0.38. What is the power of the test?

2. Find Probability of Type II Error / Power of Test

To test Ho: p = 0.30 versus H1: p ≠ 0.30, a simple random sample of n = 500 is obtained and 170 successes are observed. If the researcher decides to test this hypothesis at the α = 0.01 level of significance, compute the probability of making a Type II Error if the true population proportion is 0.34. What is the power of the test?

Finding Probability of a Type II Error / Power of Test (Mean) (TI-83/84)

1. Write down claim
2. Write down null and alternate hypothesis
3. Draw picture (determine tail(s))
4. Find ZA_L and/or ZA_R (you will have both for two tails) – round to 3 decimal places
   - Left Tail: ZA_L = INVNORM(α)
   - Right Tail: ZA_R = INVNORM(1-α)
   - Two Tails: ZA_L = INVNORM(α/2) and ZA_R = INVNORM(1-α/2)

Finding Probability of a Type II Error / Power of Test (Mean) (TI-83/84)

5. Find $\bar{x}_L$ and/or $\bar{x}_R$ (you will have both for two tails) – round to 3 decimal places

- Left Tail: $\bar{x}_L = \mu_o + ZA_L \cdot \frac{\sigma}{\sqrt{n}}$
- Right Tail: $\bar{x}_R = \mu_o + ZA_R \cdot \frac{\sigma}{\sqrt{n}}$
- Two Tails: $\bar{x}_L = \mu_o + ZA_L \cdot \frac{\sigma}{\sqrt{n}}$ and $\bar{x}_R = \mu_o + ZA_R \cdot \frac{\sigma}{\sqrt{n}}$

μ_o is the number from the claim

Finding Probability of a Type II Error / Power of Test (Mean) (TI-83/84)

6. Find z_L and/or z_R (you will have both for two tails) – round to 2 decimal places

- Left Tail: $z_L = \frac{\bar{x}_L - \mu}{\frac{\sigma}{\sqrt{n}}}$
- Right Tail: $z_R = \frac{\bar{x}_R - \mu}{\frac{\sigma}{\sqrt{n}}}$
- Two Tails: $z_L = \frac{\bar{x}_L - \mu}{\frac{\sigma}{\sqrt{n}}}$ and $z_R = \frac{\bar{x}_R - \mu}{\frac{\sigma}{\sqrt{n}}}$

μ is the true population mean

Finding Probability of a Type II Error / Power of Test (Mean) (TI-83/84)

7. Find the Probability of a Type II Error (β)

- Left Tail: β = NORMALCDF(z_L,E99)
- Right Tail: β = NORMALCDF(-E99,z_R)
- Two Tails: β = NORMALCDF(z_L,z_R)

8. Find the power of the test (1-β)

Finding Probability of a Type II Error / Power of Test (Mean) (By Hand)

1. Write down claim
2. Write down null and alternate hypothesis
3. Draw picture (determine tail(s))
4. Find ZA_L and/or ZA_R (you will have both for two tails) – Use Standard Normal Distribution table
   - Left Tail: ZA_L = Look up α
   - Right Tail: ZA_R = Look up (1-α)
   - Two Tails: ZA_L = Look up (α/2) and ZA_R = Look up (1-α/2)

Finding Probability of a Type II Error / Power of Test (Mean) (By Hand)

5. Find $\bar{x}_L$ and/or $\bar{x}_R$ (you will have both for two tails) – round to 3 decimal places

- Left Tail: $\bar{x}_L = \mu_o + ZA_L \cdot \frac{\sigma}{\sqrt{n}}$
- Right Tail: $\bar{x}_R = \mu_o + ZA_R \cdot \frac{\sigma}{\sqrt{n}}$
- Two Tails: $\bar{x}_L = \mu_o + ZA_L \cdot \frac{\sigma}{\sqrt{n}}$ and $\bar{x}_R = \mu_o + ZA_R \cdot \frac{\sigma}{\sqrt{n}}$

μ_o is the number from the claim

Finding Probability of a Type II Error / Power of Test (Mean) (By Hand)

6. Find z_L and/or z_R (you will have both for two tails) – round to 2 decimal places

- Left Tail: $z_L = \frac{\bar{x}_L - \mu}{\frac{\sigma}{\sqrt{n}}}$
- Right Tail: $z_R = \frac{\bar{x}_R - \mu}{\frac{\sigma}{\sqrt{n}}}$
- Two Tails: $z_L = \frac{\bar{x}_L - \mu}{\frac{\sigma}{\sqrt{n}}}$ and $z_R = \frac{\bar{x}_R - \mu}{\frac{\sigma}{\sqrt{n}}}$

μ is the true population mean

Finding Probability of a Type II Error / Power of Test (Mean) (By Hand)

7. Find the Probability of a Type II Error (β). Look up z scores in standard normal distribution table.

- Left Tail: $\beta = 1 - (\text{value from table based on } z_L)$
- Right Tail: $\beta = (\text{value from table based on } z_R)$
- Two Tails: $\beta = (\text{value from table based on } z_R) - (\text{value from table based on } z_L)$

8. Find the power of the test (1-β)

3. Find Probability of Type II Error / Power of Test

To test Ho: $\mu = 400$ versus H1: $\mu > 400$, a simple random sample of n = 100 is obtained. Assume the population standard deviation is 80. If the researcher decides to test this hypothesis at the $\alpha = 0.05$ level of significance, compute the probability of making a Type II Error if the true population mean is 420. What is the power of the test?