Rotation of Axes; General Form of a Conic

Identifying Conics

Except for degenerate cases:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

(a) Defines a parabola if $B^2 - 4AC = 0$

(b) Defines an ellipse (or a circle) if $B^2 - 4AC < 0$

(c) Defines a hyperbola if $B^2 - 4AC > 0$

1. Identify the graph of each equation without completing the square (Similar to p.417 #11-20, 43-52)

$$y^2 + 6y + x - 7 = 0$$

2. Identify the graph of each equation without completing the square (Similar to p.417 #11-20, 43-52)

$$4x^2 + y^2 - 16x + 6y + 3 = 0$$

3. Determine the appropriate rotation formulas to use so that the new equation contains no xy-term (Similar to p.417 #21-30)

$$x^2 + 16xy + y^2 - 5 = 0$$

Rotation Formulas

Given equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Angle of Rotation: $cot(2\theta) = \frac{A-C}{B}$

Then you can do the following substitution:

$x = x' \cos\theta - y' \sin\theta \qquad y = x' \sin\theta + y' \cos\theta$

May have to use:

$$\cos\theta = \sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\sin\theta = \sqrt{\frac{1 - \cos(2\theta)}{2}}$$

Standard Form of a Parabola

$$(x-h)^2 = 4p(y-k)$$
$$vertex = (h,k)$$
$$focus = (h,k+p)$$
$$directrix: y = k-p$$

$$(y-k)^2 = 4p(x-h)$$
$$vertex = (h,k)$$
$$focus = (h+p,k)$$
$$directrix: x = h-p$$

Standard Form of a Ellipse (a>b)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$
$$c=\sqrt{a^2-b^2}$$
$Center: (h,k)$
Major Axis: Parallel to x-axis
Length of Major Axis: 2a
Length of Minor Axis: 2b
$Foci: (h+c,k),\ (h-c,k)$
$Vertices: (h+a,k),\ (h-a,k)$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$
$$c=\sqrt{a^2-b^2}$$
$Center: (h,k)$
Major Axis: Parallel to y-axis
Length of Major Axis: 2a
Length of Minor Axis: 2b
$Foci: (h,k+c),\ (h,k-c)$
$Vertices: (h,k+a),\ (h,k-a)$

Standard Form of a Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$
$$b^2=c^2-a^2$$
$Center: (h,k)$
Transverse Axis:
Parallel to x-axis
$Foci: (h+c,k),\ (h-c,k)$
$Vertices: (h+a,k),\ (h-a,k)$
$Asymptotes: y-k=\pm\frac{b}{a}(x-h)$

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$
$$b^2=c^2-a^2$$
$Center: (h,k)$
Transverse Axis:
Parallel to y-axis
$Foci: (h,k+c),\ (h,k-c)$
$Vertices: (h,k+a),\ (h,k-a)$
$Asymptotes: y-k=\pm\frac{a}{b}(x-h)$

4. Rotate the axes so that the new equation contains no xy-term. Analyze and graph the new equation (Similar to p.418 #31-42)

$$xy=4$$

5. Rotate the axes so that the new equation contains no xy-term. Analyze and graph the new equation (Similar to p.418 #31-42)

$$x^2+16xy+y^2-3=0$$

6. Rotate the axes so that the new equation contains no xy-term. Analyze and graph the new equation (Similar to p.418 #31-42)

$$9x^2-6xy+y^2-10\sqrt{10}x-30\sqrt{10}y=0$$